Dimensional Analysis of Deformation

Since the fluid stresses due to translation may deform the bubbles, drops or deformable solid particles that make up the disperse phase, we should consider not only the parameters governing the deformation but also the consequences in terms of the translation velocity and the shape. We concentrate here on bubbles and drops in which surface tension, S, acts as the force restraining deformation. However, the reader will realize that there would exist a similar analysis for deformable elastic particles. Furthermore, the discussion will be limited to the case of *steady* translation, caused by gravity, g. Clearly the results could be extended to cover translation due to fluid acceleration by using an effective value of g as indicated in section (Nej).

The characteristic force maintaining the sphericity of the bubble or drop is given by SR. Deformation will occur when the characteristic anisotropy in the fluid forces approaches SR; the magnitude of the anisotropic fluid force will be given by $\mu_L W_\infty R$ for $W_\infty R/\nu_L \ll 1$ or by $\rho_L W_\infty^2 R^2$ for $W_\infty R/\nu_L \gg 1$. Thus defining a Weber number, $We = 2\rho_L W_\infty^2 R/S$, deformation will occur when We/Re approaches unity for $Re \ll 1$ or when We approaches unity for $Re \gg 1$. But evaluation of these parameters requires knowledge of the terminal velocity, W_∞, and this may also be a function of the shape. Thus one must start by expanding the functional relation of equation (Nei16) which determines W_∞ to include the Weber number:

$$F(Re, We, Fr) = 0 \tag{Nfb1}$$

This relation determines W_∞ where Fr is given by equations (Nei14). Since all three dimensionless coefficients in this functional relation include both W_∞ and R, it is simpler to rearrange the arguments by defining another nondimensional parameter, the Haberman-Morton number (1953), Hm, that is a combination of We, Re, and Fr but does not involve W_∞. The Haberman-Morton number is defined as

$$Hm = \frac{We^3}{Fr^2 Re^4} = \frac{g\mu_L^4}{\rho_L S^3}\left(1 - \frac{m_p}{\rho_L v}\right) \tag{Nfb2}$$

In the case of a bubble, $m_p \ll \rho_L v$ and therefore the factor in parenthesis is usually omitted. Then Hm becomes independent of the bubble size. It follows that the terminal velocity of a bubble or drop can be represented by functional relation

$$F(Re, Hm, Fr) = 0 \quad \text{or} \quad F^*(Re, Hm, C_D) = 0 \tag{Nfb3}$$

and we shall confine the following discussion to the nature of this relation for bubbles ($m_p \ll \rho_L v$).

Some values for the Haberman-Morton number (with $m_p/\rho_L v = 0$) for various saturated liquids are shown in figure 1; other values are listed in table 1. Note that for all but the most viscous liquids, Hm is much less than unity. It is, of course, possible to have fluid accelerations much larger than g; however, this is unlikely to cause Hm values greater than unity in practical multiphase flows of most liquids.

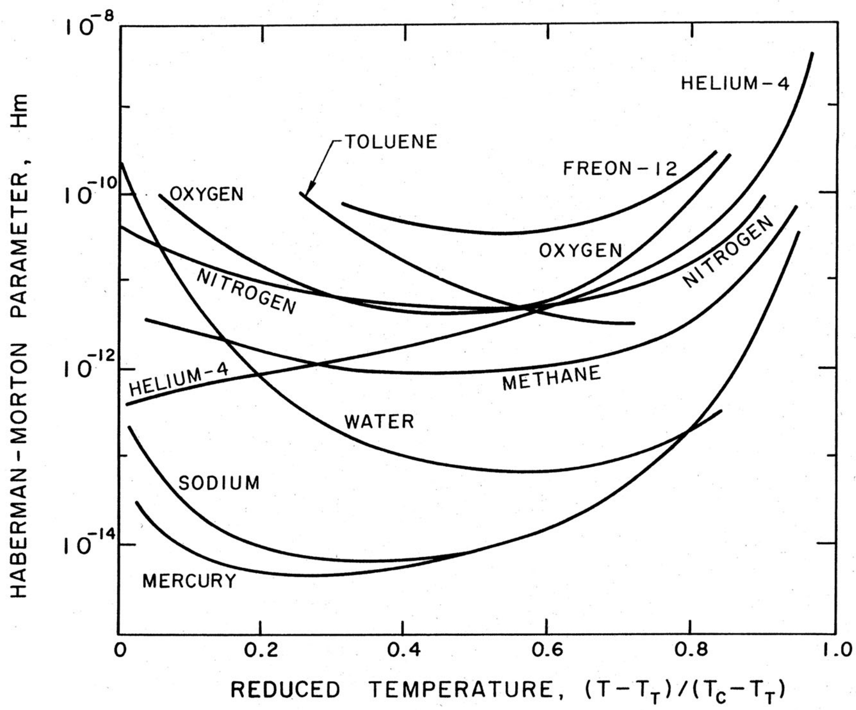

Figure 1: Values of the Haberman-Morton parameter, Hm, for various pure substances as a function of reduced temperature where T_T is the triple point temperature and T_C is the critical point temperature.

Table 1: Values of the Haberman-Morton numbers, $Hm = g\mu_L^4/\rho_L S^3$, for various liquids at normal temperatures.

Filtered Water	0.25×10^{-10}	Turpentine	2.41×10^{-9}
Methyl Alcohol	0.89×10^{-10}	Olive Oil	7.16×10^{-3}
Mineral Oil	1.45×10^{-2}	Syrup	0.92×10^{6}