## Spray Formation by Initially Laminar Jets

In many important technological processes, sprays are formed by the breakup of a liquid jet injected into a gaseous atmosphere. One of the most important of these, is fuel injection in power plants, aircraft and automobile engines and here the character of the spray formed is critical not only for performance but also for pollution control. Consequently much effort has gone into the design of the nozzles (and therefore the jets) that produce sprays with desirable characteristics. *Atomizing* nozzles are those that produce particularly fine sprays. Other examples of technologies in which there is a similar focus on the nature of the spray produced are ink-jet printing and the *scrubbing* of exhaust gases to remove particulate pollutants.



Figure 1: Photographs of an initially laminar jet emerging from a nozzle. The upper photograph shows the instability wave formation and growth and the lower shows the spray droplet formation at a location 4 diameters further downstream. The lower photograph shows the same jet even further downstream. Reproduced from Hoyt and Taylor (1977b) with the permission of the authors.

Because of its technological importance, we focus here on the circumstance in which the jet is turbulent when it emerges from the nozzle. However, in passing, we note that the breakup of laminar jets may also be of interest. Two photographs of initially laminar jets taken by Hoyt and Taylor (1977a,b) are reproduced in figure 1. Photographs such as the upper one clearly show that transition to turbulence occurs because the interfacial layer formed when the liquid boundary layer leaves the nozzle becomes unstable. The Tollmein-Schlicting waves (remarkably two-dimensional) exhibit a well-defined wavelength and grow to non-linear amplitudes at which they breakup to form droplets in the gas. Sirignano and Mehring (2000) provide a review of the extensive literature on linear and non-linear analyses of the stability of liquid jets, not only round jets but also planar and annular jets. The author (Brennen 1970) examined the development of interfacial instability waves in the somewhat different context of cavity flows; this analysis demonstrated that the appropriate length scale is the thickness of the internal boundary layer,  $\delta$ , on the nozzle walls at the point where the free surface detaches. This is best characterized by the momentum thickness,  $\delta_2$ , though other measures of the boundary layer thickness have also been used. The stability analysis yields the most unstable wavelength for the Tollmein-Schlichting waves (normalized by  $\delta_2$ ) as a function of the Reynolds number of the interfacial boundary layer (based on the jet velocity and  $\delta_2$ ). At larger Reynolds number, the ratio of wavelength to  $\delta_2$  reaches an asymptotic value of about 25, independent of Reynolds number. Brennen (1970) and Hoyt and Taylor (1977a,b) observe that these predicted wavelengths are in accord with those observed.

A natural extension of this analysis is to argue that the size of the droplets formed by the non-linear breakup of the instability waves will scale with the wavelength of those waves. Indeed, the pictures of Hoyt and Taylor (1977a,b) exemplified by the lower photograph in figure 1 suggest that this is the case. It follows that at higher Reynolds numbers, the droplet size should scale with the boundary layer thickness,  $\delta_2$ . Wu, Miranda and Faeth (1995) have shown that this is indeed the case for the initial drop formation in initially nonturbulent jets.

Further downstream the turbulence spreads throughout the core of the jet and the subsequent jet breakup and droplet formation is then similar to that of jets that are initially turbulent. We now turn to that circumstance.