Unsteady Particle Motions

Having reviewed the steady motion of a particle relative to a fluid, we must now consider the consequences of unsteady relative motion in which either the particle or the fluid or both are accelerating. The complexities of fluid acceleration are delayed until the next section. First we shall consider the simpler circumstance in which the fluid is either at rest or has a steady uniform streaming motion (U = constant) far from the particle. Clearly the second case is readily reduced to the first by a simple Galilean transformation and it will be assumed that this has been accomplished.

In the ideal case of unsteady inviscid potential flow, it can then be shown by using the concept of the total kinetic energy of the fluid that the force on a rigid particle in an incompressible flow is given by F_i, where

$$F_i = -M_{ij} \frac{dV_j}{dt} \tag{Nee1}$$

where M_{ij} is called the *added mass matrix* (or tensor) though the name *induced inertia tensor* used by Batchelor (1967) is, perhaps, more descriptive. The reader is referred to Sarpkaya and Isaacson (1981), Yih (1969), or Batchelor (1967) for detailed descriptions of such analyses. The above mentioned methods also show that M_{ij} for any finite particle can be obtained from knowledge of several *steady* potential flows. In fact,

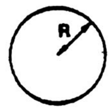
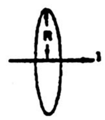
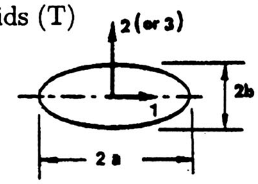
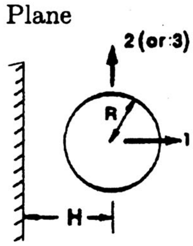
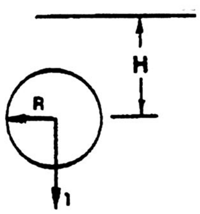
$$M_{ij} = \frac{\rho_C}{2} \int_{\substack{volume \\ of\ fluid}} u_{ik} u_{jk} \, d(volume) \tag{Nee2}$$

where the integration is performed over the entire volume of the fluid. The velocity field, u_{ij}, is the fluid velocity in the i direction caused by the *steady* translation of the particle with unit velocity in the j direction. Note that this means that M_{ij} is necessarily a symmetric matrix. Furthermore, it is clear that particles with planes of symmetry will not experience a force perpendicular to that plane when the direction of acceleration is parallel to that plane. Hence if there is a plane of symmetry perpendicular to the k direction, then for $i \neq k$, $M_{ki} = M_{ik} = 0$, and the only off-diagonal matrix elements that can be nonzero are M_{ij}, $j \neq k$, $i \neq k$. In the special case of the sphere *all* the off-diagonal terms will be zero.

Tables of some available values of the diagonal components of M_{ij} are given by Sarpkaya and Isaacson (1981) who also summarize the experimental results, particularly for planar flows past cylinders. Other compilations of added mass results can be found in Kennard (1967), Patton (1965), and Brennen (1982). Some typical values for three-dimensional particles are listed in the table. The uniform diagonal value for a sphere (often referred to simply as the added mass of a sphere) is $2\rho_C \pi R^3/3$ or one-half the displaced mass of fluid. This value can readily be obtained from equation (Nee2) using the steady flow results given in equations (Neb5) to (Neb8). In general, of course, there is *no* special relation between the added mass and the displaced mass. Consider, for example, the case of the infinitely thin plate or disc with zero displaced mass which has a finite added mass in the direction normal to the surface. Finally, it should be noted that the literature contains little, if any, information on off-diagonal components of added mass matrices.

Now consider the application of these potential flow results to real viscous flows at high Reynolds numbers (the case of low Reynolds number flows will be discussed in section (Neh)). Significant doubts about the applicability of the added masses calculated from potential flow analysis would be justified because of the experience of D'Alembert's paradox for steady potential flows and the substantial difference between the streamlines of the potential and actual flows. Furthermore, analyses of experimental results will require the

Table. Added masses (diagonal terms in M_{ij}) for some three-dimensional bodies (particles): (T) Potential flow calculations, (E) Experimental data from Patton (1965).

Particle		Matrix Element	Value
Sphere (T)		M_{ii}	$\frac{2}{3}\rho_C \pi R^3$
Disc (T)		M_{11}	$\frac{8}{3}\rho_C R^3$
Ellipsoids (T)		M_{ii} $= K_{ii}\frac{4}{3}\rho_C \pi a b^2$	a/b K_{11} $K_{22}(K_{33})$ 2 0.209 0.702 5 0.059 0.895 10 0.021 0.960
Sphere near Plane Wall (T) $(R/H \ll 1)$		M_{ii} $= K_{ii}\frac{4}{3}\rho_C \pi R^3$	$K_{11} = \frac{1}{2}\left(1 + \frac{3}{8}\frac{R^3}{H^3} + \ldots\right)$ $K_{22} = \frac{1}{2}\left(1 + \frac{3}{16}\frac{R^3}{H^3} + \ldots\right)$ $K_{33} = K_{22}$
Sphere near Free Surface (E) $(R/H \ll 1)$		M_{ii} $= K_{ii}\frac{4}{3}\rho_C \pi R^3$	H/R K_{11} 8.0 0.52 4.0 0.59 2.0 0.54 1.0 0.44 0.0 0.25

separation of the *added mass* forces from the viscous drag forces. Usually this is accomplished by heuristic summation of the two forces so that

$$F_i = -M_{ij}\frac{dV_j}{dt} - \frac{1}{2}\rho_C A C_{ij}|V_j|V_j \qquad \text{(Nee3)}$$

where C_{ij} is a lift and drag coefficient matrix and A is a typical cross-sectional area for the body. This is known as Morison's equation (see Morison *et al.* 1950).

Actual unsteady high Reynolds number flows are more complicated and not necessarily compatible with such simple superposition. This is reflected in the fact that the coefficients, M_{ij} and C_{ij}, appear from the experimental results to be not only functions of Re but also functions of the reduced time or frequency of the unsteady motion. Typically experiments involve either oscillation of a body in a fluid or acceleration from rest. The most extensively studied case involves planar flow past a cylinder (for example, Keulegan and Carpenter 1958), and a detailed review of this data is included in Sarpkaya and Isaacson (1981). For oscillatory motion of the cylinder with velocity amplitude, U_M, and period, t^*, the coefficients are functions of both the Reynolds number, $Re = 2U_M R/\nu_C$, and the reduced period or Keulegan-Carpenter number, $Kc = U_M t^*/2R$. When the amplitude, $U_M t^*$, is less than about $10R$ ($Kc < 5$), the inertial effects dominate and M_{ii} is only a little less than its potential flow value over a wide range of Reynolds numbers

($10^4 < Re < 10^6$). However, for larger values of Kc, M_{ii} can be substantially smaller than this and, in some range of Re and Kc, may actually be negative. The values of C_{ii} (the drag coefficient) that are deduced from experiments are also a complicated function of Re and Kc. The behavior of the coefficients is particularly pathological when the reduced period, Kc, is close to that of vortex shedding (Kc of the order of 10). Large transverse or *lift* forces can be generated under these circumstances. To the author's knowledge, detailed investigations of this kind have not been made for a spherical body, but one might expect the same qualitative phenomena to occur.