Batch Sedimentation

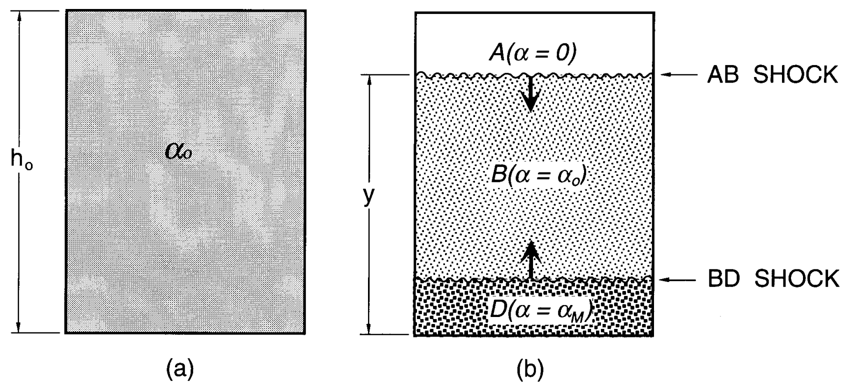

Figure 1: Type I batch sedimentation.

Since it presents a useful example of kinematic shock propagation, we shall consider the various phenomena that occur in batch sedimentation. For simplicity, it is assumed that this process begins with a uniform suspension of solid particles of volume fraction, α_0, in a closed vessel (figure 1(a)). Conceptually, it is convenient to visualize gravity being switched on at time, $t = 0$. Then the sedimentation of the particles leaves an expanding clear layer of fluid at the top of the vessel as indicated in figure 1(b). This implies that at time $t = 0$ a kinematic shock is formed at the top of the vessel. This shock is the moving boundary between the region A of figure 1(b) in which $\alpha = 0$ and the region B in which $\alpha = \alpha_0$. It travels downward at the shock propagation speed given by the slope of the line AB in figure 2(left) (note that in this example $j = 0$).

Now consider the corresponding events that occur at the bottom of the vessel. Beginning at time $t = 0$, particles will start to come to rest on the bottom and a layer comprising particles in a packed state at $\alpha = \alpha_m$ will systematically grow in height (we neglect any subsequent adjustments to the packing that might occur as a result of the increasing overburden). A kinematic shock is therefore present at the

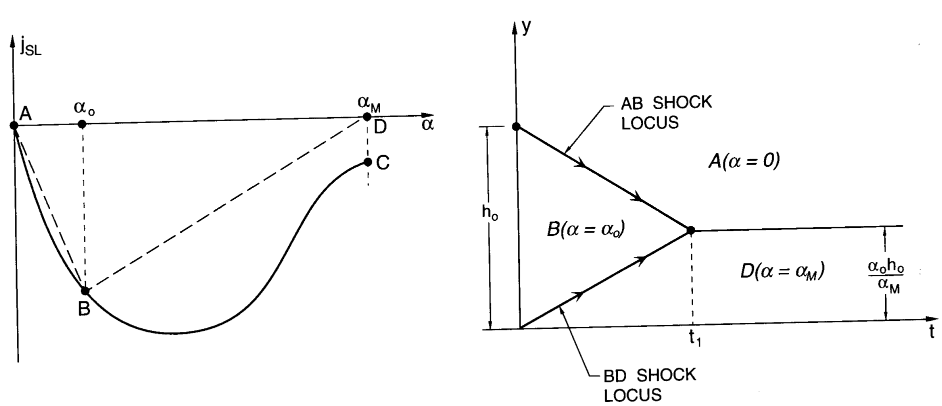

Figure 2: Drift flux chart and sedimentation evolution diagram for Type I batch sedimentation.

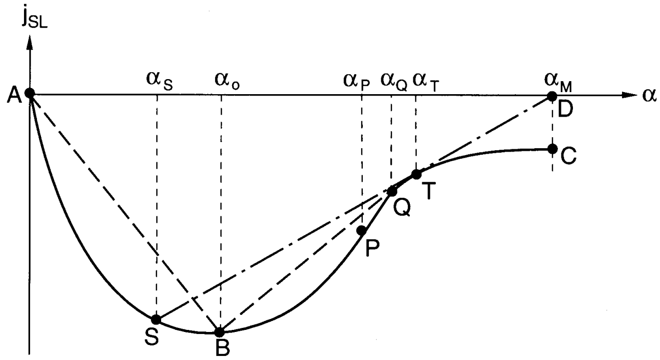

Figure 3: Drift flux chart for Type III sedimentation.

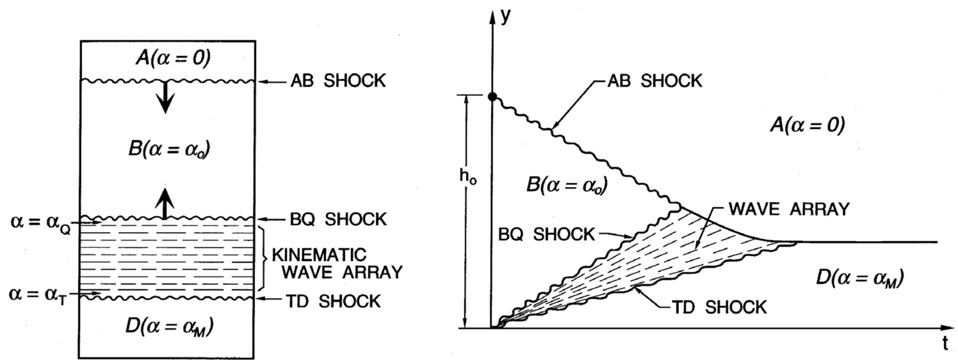

Figure 4: Sketch and evolution diagram for Type III sedimentation.

interface between the packed region D (figure 1(b)) and the region B; clearly this shock is also formed at the bottom at time $t = 0$ and propagates upward. Since the conditions in the packed bed are such that both the particle and liquid flux are zero and, therefore, the drift flux is zero, this state is represented by the point D in the drift flux chart, figure 2(left) (rather than the point C). It follows that, provided that none of the complications discussed later occur, the propagation speed of the upward moving shock is given by the slope of the line BD in figure 2(left). Note that both the downward moving AB shock and the upward moving BD shock are stable.

The progress of the batch sedimentation process can be summarized in a time evolution diagram such as figure 2(right) in which the elevations of the shocks are plotted as a function of time. When the AB and BD shocks meet at time $t = t_1$, the final packed bed depth equal to $\alpha_0 h_0/\alpha_m$ is achieved and the sedimentation process is complete. Note that

$$t_1 = \frac{h_0\alpha_0(\alpha_m - \alpha_0)}{\alpha_m j_{SL}(\alpha_0)} = \frac{h_0(\alpha_m - \alpha_0)}{\alpha_m(1 - \alpha_0)u_{SL}(\alpha_0)} \qquad \text{(Nsh1)}$$

The simple batch evolution described above is known as Type I sedimentation. There are, however, other complications that can arise if the shape of the drift flux curve and the value of α_0 are such that the line connecting B and D in figure 2(left) intersects the drift flux curve. Two additional types of sedimentation may occur under those circumstances and one of these, Type III, is depicted in figures 3 and 4. In figure 3, the line STD is tangent to the drift flux curve at the point T and the point P is the point of inflection in the drift flux curve. Thus are the volume fractions, α_S, α_P and α_T defined. If α_0 lies between α_S and α_P the process is known as Type III sedimentation and this proceeds as follows (the line BQ is a tangent to the drift flux curve at the point Q and defines the value of α_Q). The first shock to form at the bottom is one in which the volume fraction is increased from α_0 to α_Q. As depicted in figure 4 this is followed by a continuous array of small kinematic waves through which the volume fraction is increased from α_Q to α_T. Since the speeds of these waves are given by the slopes of the drift flux curve at the appropriate volume fractions, they travel progressively more slowly than the initial BQ shock. Finally this kinematic wave array is followed by a second, upward moving shock, the TD shock across which the volume fraction increases from α_Q to α_m. While this package of waves is rising from the bottom, the usual AB shock is moving down from the top. Thus, as depicted in figure 4, the sedimentation process is more complex but, of course, arrives at the same final state as in Type I.

A third type, Type II, occurs when the initial volume fraction, α_0, is between α_P and α_T. This evolves in a manner similar to Type III except that the kinematic wave array is not preceded by a shock like the BQ shock in Type III.