Numerical Methods

With the modern evolution of computational methods it has become increasingly viable to consider more direct numerical methods for the solution of free surface flows, even in circumstances in which analytical solutions could be generated. It would be beyond the scope of this text to survey these computational methods, and so we confine our discussion to some brief comments on the methods used in the past. These can be conveniently divided into two types. Some of the literature describes "field" methods in which the entire flow field is covered by a lattice of grids and node points at which the flow variables are evaluated. But most of the work in the past has focused on the use of "boundary element" methods that make use of superposition of the fundamental singularity solutions for potential flows. A few methods do not fit into these categories; for example, the expansion technique devised by Garabedian (1956) in order to construct axisymmetric flow solutions from the corresponding planar flows.

Methods for the synthesis of potential flows using distributed singularities can, of course, be traced to the original work of Rankine (1871). The first attempts to use distributions of sources and sinks to find solutions to axisymmetric cavity flow problems appear to have been made by Reichardt and Munzner (1950). They distributed doublets on the axis and sought symmetric, Rankine-like body shapes with nearly constant surface pressure except for fore and aft caps in order to simulate Riabouchinsky flows. The problem with this approach is its inability to model the discontinuous or singular behavior at the free surface detachment points. This requires a distribution of surface singularities that can either be implemented explicitly (most conveniently with surface vortex sheet elements) or by the equivalent use of Green's function methods as pioneered by Trefftz (1916) in the context of jets. Distributions of surface singularities to model cavity flows were first employed by Landweber (1951), Armstrong and Dunham (1953), and Armstrong and Tadman (1954). The latter used these methods to generate solutions for the axisymmetric Riabouchinsky solutions of cavitating discs and spheres. The methods were later extended to three-dimensional potential flows by Struck (1966), who addressed the problem of an axisymmetric body at a small angle of attack to the oncoming stream.

As computational capacity grew, it became possible to examine more complex three-dimensional flows and lifting bodies using boundary element methods. For example, Lemonnier and Rowe (1988) computed solutions for a partially cavitating hydrofoil and Uhlman (1987, 1989) has generated solutions for hydrofoils with both partial cavitation and supercavitation. These methods solve for the velocity. The position of the cavity boundary is determined by an iterative process in which the dynamic condition is satisfied on an approximate cavity surface and the kinematic condition is used to update the location of the surface. More recently, a method that uses Green's theorem to solve for the potential has been developed by Kinnas and Fine (1990) and has been applied to both partially and supercavitating hydrofoils. This appears to be superior to the velocity-based methods in terms of convergence.

Efforts have also been made to develop "field" methods for cavity flows. Southwell and Vaisey (1946) (see also Southwell 1948) first explored the use of relaxation methods to solve free surface problems but did not produce solutions for any realistic cavity flows. Woods (1951) suggested that solutions to axisymmetric cavity flows could be more readily obtained in the geometrically simpler (ϕ, ψ) plane, and Brennen (1969a) used this suggestion to generate Riabouchinsky model solutions for a cavitating disc and sphere in a finite water tunnel. In more recent times, it has become clear that boundary integral methods are more efficient for potential flows. However, field methods must still be used when seeking solutions to the more complete viscous flow problem. Significant progress has been made in the last few years in developing Navier-

Stokes solvers for free surface problems in general and cavity flow problems in particular (see, for example, Deshpande *et al.* 1993).