Shock Wave Structure

Bubble dynamics do not affect the results presented thus far since the speed, u_1, depends only on the equilibrium conditions upstream and downstream. However, the existence and structure of the shock depend on the bubble dynamic terms in equation (Nme5). That equation is more conveniently written in terms of a radius ratio, $r = R/R_1$, and a dimensionless coordinate, $z = x/R_1$:

$$\left(1-\alpha_1+\alpha_1 r^3\right)^2 r\frac{d^2r}{dz^2}+\frac{3}{2}\left(1-\alpha_1+\alpha_1 r^3\right)\left(1-\alpha_1+3\alpha_1 r^3\right)\left(\frac{dr}{dz}\right)^2$$

$$+\left(1-\alpha_1+\alpha_1 r^3\right)\frac{4\nu_L}{u_1R_1}\frac{1}{r}\frac{dr}{dz}+\alpha_1\left(1-\alpha_1\right)\left(1-r^3\right)$$

$$=\frac{1}{u_1^2}\left[\frac{(p_1-p_V)}{\rho_L}\left(r^{-3k}-1\right)+\frac{2S}{\rho_L R_1}\left(r^{-3k}-r^{-1}\right)\right] \quad \text{(Nmf1)}$$

It could also be written in terms of the void fraction, α, since

$$r^3=\frac{\alpha}{(1-\alpha)}\frac{(1-\alpha_1)}{\alpha_1} \quad \text{(Nmf2)}$$

When examined in conjunction with the expression in equation (Nme8) for u_1, it is clear that the solution, $r(z)$ or $\alpha(z)$, for the structure of the shock is a function only of α_1, α_2, k, $R_1(p_1-p_V)/S$, and the effective Reynolds number, u_1R_1/ν_L, where, as previously mentioned, ν_L should incorporate the various forms of bubble damping.

Equation (Nmf1) can be readily integrated numerically and typical solutions are presented in figure 1 for $\alpha_1 = 0.3$, $k = 1.4$, $R_1(p_1-p_V)/S \gg 1$, $u_1R_1/\nu_L = 100$, and two downstream volume fractions, $\alpha_2 = 0.1$ and 0.05. These examples illustrate several important features of the structure of these shocks. First, the initial collapse is followed by many rebounds and subsequent collapses. The decay of these nonlinear

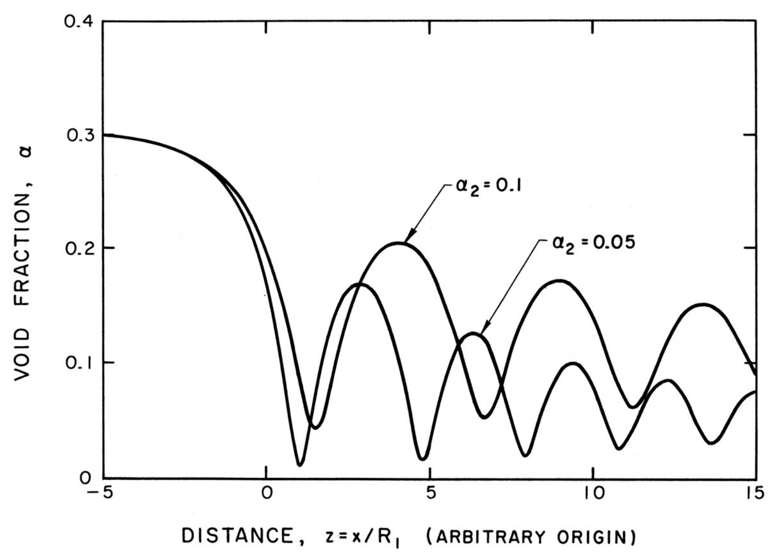

Figure 1: The typical structure of a shock wave in a bubbly mixture is illustrated by these examples for $\alpha_1 = 0.3$, $k = 1.4$, $R_1(p_1-p_V)/S \gg 1$, and $u_1R_1/\nu_L = 100$.

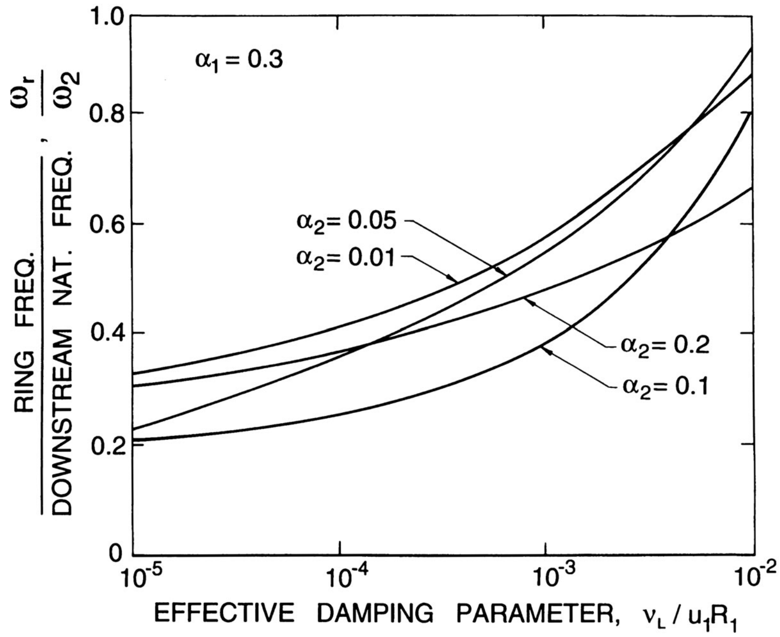

Figure 2: The ratio of the ring frequency downstream of a bubbly mixture shock to the natural frequency of the bubbles far downstream as a function of the effective damping parameter, ν_L/u_1R_1, for $\alpha_1 = 0.3$ and various downstream void fractions as indicated.

oscillations is determined by the damping or u_1R_1/ν_L. Though u_1R_1/ν_L includes an effective kinematic viscosity to incorporate other contributions to the bubble damping, the value of u_1R_1/ν_L chosen for this example is probably smaller than would be relevant in many practical applications, in which we might expect the decay to be even smaller. It is also valuable to identify the nature of the solution as the damping is eliminated ($u_1R_1/\nu_L \rightarrow \infty$). In this limit the distance between collapses increases without bound until the structure consists of one collapse followed by a downstream asymptotic approach to a void fraction of α_1 (*not* α_2). In other words, no solution in which $\alpha \rightarrow \alpha_2$ exists in the absence of damping.

Another important feature in the structure of these shocks is the typical interval between the downstream oscillations. This *ringing* will, in practice, result in acoustic radiation at frequencies corresponding to this interval, and it is of importance to identify the relationship between this ring frequency and the natural frequency of the bubbles downstream of the shock. A characteristic ring frequency, ω_r, for the shock oscillations can be defined as

$$\omega_r = 2\pi u_1/\Delta x \tag{Nmf3}$$

where Δx is the distance between the first and second bubble collapses. The natural frequency of the bubbles far downstream of the shock, ω_2, is given by (see equation (Nmc10))

$$\omega_2^2 = \frac{3k(p_2 - p_V)}{\rho_L R_2^2} + (3k-1)\frac{2S}{\rho_L R_2^3} \tag{Nmf4}$$

and typical values for the ratio ω_r/ω_2 are presented in figure 2 for $\alpha_1 = 0.3$, $k = 1.4$, $R_1(p_1 - p_V)/S \gg 1$, and various values of α_2. Similar results were obtained for quite a wide range of values of α_1. Therefore note that the frequency ratio is primarily a function of the damping and that ring frequencies up to a factor of 10 less than the natural frequency are to be expected with typical values of the damping in water. This reduction in the typical frequency associated with the collective behavior of bubbles presages the natural frequencies of bubble clouds, that are discussed in the next section.

While the focus in the preceding two sections has been on normal shock waves, the analysis can be generalized to cover oblique shocks. Figure 3 is a photograph taken in a supersonic bubbly tunnel (Eddington

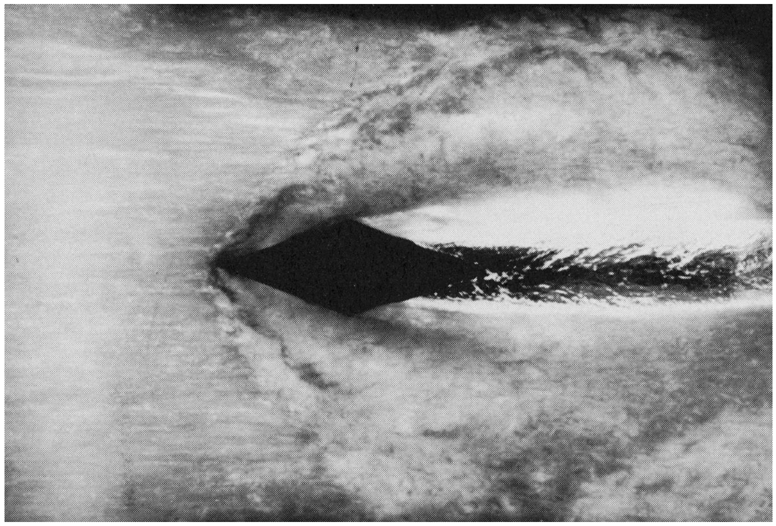

Figure 3: Supersonic bubbly flow past a 20° half-angle wedge at a Mach number of 4. Flow is from left to right. Photograph taken in supersonic bubbly flow tunnel (Eddington 1967) and reproduced with permission.

1967) and shows a Mach 4 flow past a 20° half-angle wedge. The oblique bow shock waves are clearly evident and one can also detect some of the structure of the shocks.