Averaging

In the section (Nac) it was implicitly assumed that there existed an *infinitesimal* volume of dimension, ϵ, such that ϵ was not only very much smaller than the typical distance over which the flow properties varied significantly but also very much larger than the size of the individual phase elements (the disperse phase particles, drops or bubbles). The first condition is necessary in order to define derivatives of the flow properties within the flow field. The second is necessary in order that each *averaging* volume (of volume ϵ^3) contain representative samples of each of the components or phases. In the sections that follow (sections (Nbb) to (Nbi)), we proceed to develop the effective differential equations of motion for multiphase flow assuming that these conditions hold.

However, one of the more difficult hurdles in treating multiphase flows, is that the above two conditions are rarely both satisfied. As a consequence the averaging volumes contain a finite number of finite-sized particles and therefore flow properties such as the continuous phase velocity vary significantly from point to point within these averaging volumes. These variations pose the challenge of how to define appropriate average quantities in the averaging volume. Moreover, the gradients of those averaged flow properties appear in the equations of motion that follow and the mean of the gradient is not necessarily equal to the gradient of the mean. These difficulties will be addressed after we have explored the basic structure of the equations in the absence of such complications.