## An Internet Book on Fluid Dynamics

## Frictional Loss in Disperse Horizontal Flow

We begin with a discussion of disperse horizontal flow. There exists a substantial body of data relating to the frictional losses or pressure gradient, $(-d p / d s)$, in a straight pipe of circular cross-section (the coordinate $s$ is measured along the axis of the pipe). Clearly $(-d p / d s)$ is a critical factor in the design of many systems, for example slurry pipelines. Therefore a substantial data base exists for the flows of mixtures of solids and water in horizontal pipes. The hydraulic gradient is usually non-dimensionalized using the pipe diameter, $d$, the density of the suspending phase ( $\rho_{L}$ if liquid), and either the total volumetric flux, $j$, or the volumetric flux of the suspending fluid ( $j_{L}$ if liquid). Thus, commonly used friction coefficients are

$$
\begin{equation*}
C_{f}=\frac{d}{2 \rho_{L} j_{L}^{2}}\left(-\frac{d p}{d s}\right) \quad \text { or } \quad C_{f}=\frac{d}{2 \rho_{L} j^{2}}\left(-\frac{d p}{d s}\right) \tag{Nkb1}
\end{equation*}
$$

and, in parallel with the traditional Moody diagram for single phase flow, these friction coefficients are usually presented as functions of a Reynolds number for various mixture ratios as characterized by the volume fraction, $\alpha$, or the volume quality, $\beta$, of the suspended phase. Commonly used Reynolds numbers are based on the pipe diameter, the viscosity of the suspending phase ( $\nu_{L}$ if liquid) and either the total volumetric flux, $j$, or the volumetric flux of the suspending fluid.

For a more complete review of slurry pipeline data the reader is referred to Shook and Roco (1991) and Lazarus and Neilsen (1978). For the solids/gas flows associated with the pneumatic conveying of solids, Soo (1983) provides a good summary. For boiling flows or for gas/liquid flows, the reader is referred to the reviews of Hsu and Graham (1976) and Collier and Thome (1994).

The typical form of the friction coefficient data is illustrated in figures 1 and 2 taken from Lazarus and Neilson (1978). Typically the friction coefficient increases markedly with increasing concentration and this increase is more significant the lower the Reynolds number. Note that the measured increases in the friction


Figure 1: Typical friction coefficients (based on the liquid volumetric flux and the liquid density) plotted against Reynolds number (based on the liquid volumetric flux and the liquid viscosity) for the horizontal pipeline flow $(d=5.2 \mathrm{~cm})$ of sand ( $D=0.018 \mathrm{~cm}$ ) and water at $21^{\circ} \mathrm{C}$ (Lazarus and Neilson 1978).


Figure 2: Typical friction coefficients (based on the liquid volumetric flux and the liquid density) plotted against Reynolds number (based on the liquid volumetric flux and the liquid viscosity) for the horizontal pipeline flow of four different solid/liquid mixtures (Lazarus and Neilson 1978).
coefficient can exceed an order of magnitude. For a given particle size and density, the flow in a given pipe becomes increasingly homogeneous as the flow rate is increased since, as discussed in section ( Njf ), the typical mixing velocity is increasing while the typical segregation velocity remains relatively constant. The friction coefficient is usually increased by segregation effects, so, for a given pipe and particles, part of the decrease in the friction coefficient with increasing flow rate is due to the normal decrease with Reynolds number and part is due to the increasing homogeneity of the flow. Figure 2, taken from Lazarus and Neilson, shows how the friction coefficient curves for a variety of solid-liquid flows, tend to asymptote at higher Reynolds numbers to a family of curves (shown by the dashed lines) on which the friction coefficient is a function only of the Reynolds number and volume fraction. These so-called base curves pertain when the flow is sufficiently fast for complete mixing to occur and the flow regime becomes homogeneous. We first address these base curves and the issue of homogeneous flow friction. Later, in section (Nkd), we comment on the departures from the base curves that occur at lower flow rates when the flow is in the heterogeneous or saltation regimes.

