Bubble/Flow Interactions

The maximum-modulus theorem states that maxima of a harmonic function must occur on the boundary and not in the interior of the region of solution of that function (see, for example, Titchmarsh 1947). Consequently, a pressure minimum in a steady, inviscid, potential flow must occur on the boundary of that flow (see Kirchhoff 1869, Birkhoff and Zarantonello 1957). Moreover, real fluid effects in many flows do not alter the fact that the minimum pressure occurs at or close to a solid surface. Perhaps the most common exception to this rule is in vortex cavitation, where the unsteady effects and/or viscous effects associated with vortex shedding or turbulence cause deviation from the maximum-modulus theorem; but discussion of this type of cavitation is delayed until later. In the many flows in which the minimum pressure does occur on a boundary, it follows that the cavitation bubbles that form in the vicinity of that point are likely to be affected by and to interact with that boundary, which we will assume is a solid surface. We observe, furthermore, that any curvature of the solid surface or, more specifically, of the streamlines in the vicinity of the minimum pressure point will cause pressure gradients normal to the surface, which are often substantially larger than those in the streamwise direction. These normal pressure gradients will force the bubble toward the surface and may cause substantial departure from sphericity. Consequently, even before boundary layer effects are factored into the picture, it is evident that the dynamics of individual cavitation bubbles may be significantly altered by interactions with the nearby solid surface and the flow near that surface. In this section we focus attention on these bubble/wall or bubble/flow interactions (grouped together in the term bubble/flow interactions).

Before describing some of the experimental observations of bubble/flow interactions, it is valuable to consider the relative sizes of the cavitation bubbles and the viscous boundary layer. In the flow of a uniform stream of velocity, U, around an object such as a hydrofoil with typical dimension, ℓ, the thickness of the laminar boundary layer near the minimum pressure point will be given qualitatively by $\delta = (\nu_L \ell / U)^{\frac{1}{2}}$. Parenthetically, we note that transition to turbulence usually occurs downstream of the point of minimum pressure, and consequently the appropriate boundary layer thickness for limited cavitation confined to the immediate neighborhood of the low-pressure region is the laminar boundary layer thickness. Moreover, the approximate analysis of section (Nhb) yields a typical maximum bubble radius, R_M, given by

$$R_M \approx 2\ell(-\sigma - C_{pmin}) \tag{Ntc1}$$

It follows that the ratio of the boundary layer thickness to the maximum bubble radius, δ / R_M, is roughly given by

$$\frac{\delta}{R_M} = \frac{1}{2(-\sigma - C_{pmin})} \left\{ \frac{\nu_L}{\ell U} \right\}^{\frac{1}{2}} \tag{Ntc2}$$

Therefore, provided $(-\sigma - C_{pmin})$ is of the order of 0.1 or greater, it follows that for the high Reynolds numbers, $U\ell / \nu_L$, which are typical of most of the flows in which cavitation is a problem, the boundary layer is usually much thinner than the typical dimension of the bubble. This does not mean the boundary layer is unimportant. But we can anticipate that those parts of the cavitation bubble farthest from the solid surface will interact with the primarily inviscid flow outside the boundary layer, while those parts close to the solid surface will be affected by the boundary layer.