Leidenfrost Effect

To analyze the Leidenfrost effect, we assume the simple geometry shown in figure 1 in which a thin, uniform layer of vapor of thickness δ separates the hemispherical droplet (radius, R) from the wall. The droplet is assumed to have been heated to the saturation temperature T_e and the temperature difference $T_w - T_e$ is denoted by ΔT. Then the heat flux per unit surface area across the vapor layer is given by $k_V \Delta T/\delta$ and this causes a mass rate of evaporation of liquid at the droplet surface of $k_V \Delta T/\delta\mathcal{L}$. The outward radial velocity of vapor at a radius of r from the center of the vapor layer, $u(r)$ (see figure 1) must match the total rate of volume production of vapor inside this radius, $\pi r^2 k_V \Delta T/\rho_V \delta \mathcal{L}$. Assuming that we use mean values of the quantities k_V, ρ_V, $\mathcal{L}$ or that these do not vary greatly within the flow, this implies that the value of u averaged over the layer thickness must be given by

$$u(r) = \frac{k_V \Delta T}{2\rho_V \mathcal{L}} \frac{r}{\delta^2} \qquad \text{(Nie1)}$$

This connects the velocity $u(r)$ of the vapor to the thickness δ of the vapor layer. A second relation between these quantities is obtained by considering the equation of motion for the viscous outward radial flow of vapor (assuming the liquid velocities are negligible). This is simply a radial Poiseuille flow in which the mean velocity across the gap, $u(r)$, must be given by

$$u(r) = -\frac{\delta^2}{12\mu_V} \frac{dp}{dr} \qquad \text{(Nie2)}$$

where $p(r)$ is the pressure distribution in the vapor layer. Substituting for $u(r)$ from equation (Nie1) and integrating we obtain the pressure distribution in the vapor layer:

$$p(r) = p_a + \frac{3k_V \mu_V \Delta T}{\rho_V \mathcal{L}} \frac{(R^2 - r^2)}{2\delta^4} \qquad \text{(Nie3)}$$

where p_a is the surrounding atmospheric pressure. Integrating the pressure difference, $p(r) - p_a$, to find the total upward force on the droplet and equating this to the difference between the weight of the droplet and the buoyancy force, $2\pi(\rho_L - \rho_V)R^3/3$, yields the following expression for the thickness, δ, of the vapor layer:

$$\frac{\delta}{R} = \left[\frac{9k_V \mu_V \Delta T}{8\rho_V(\rho_L - \rho_V) g \mathcal{L} R^3} \right]^{\frac{1}{4}} \qquad \text{(Nie4)}$$

Substituting this result back into the expression for the velocity and then evaluating the mass flow rate of vapor and consequently the rate of loss of mass of the droplet one can find the following expression for the

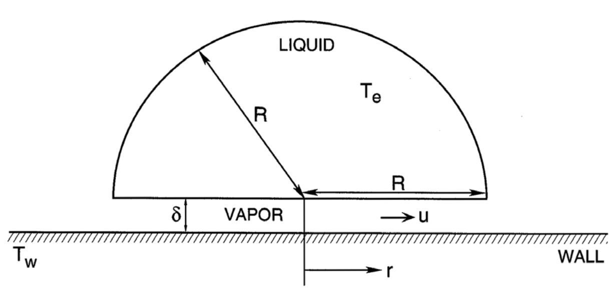

Figure 1: Hemispherical model of liquid drop for the Leidenfrost analysis.

lifetime, t_t, of a droplet of initial radius, R_o:

$$t_t = 4 \left[\frac{2\mu_V}{9\rho_V g} \right]^{\frac{1}{4}} \left[\frac{(\rho_L - \rho_V)\mathcal{L} R_o}{k_V \Delta T} \right]^{\frac{3}{4}} \tag{Nie5}$$

As a numerical example, a water droplet with a radius of $2mm$ at a saturated temperature of about $400K$ near a wall with a temperature of $500K$ will have a film thickness of just $40\mu m$ but a lifetime of just over $1hr$. Note that as ΔT, k_V or g go up the lifetime goes down as expected; on the other hand increasing R_o or μ_V has the opposite effect.