Linear Cascade Analyses

The fluid mechanics of a linear cascade will now be examined in more detail, so that the role played by the geometry of the blades and information on the resulting forces on individual blades may be used to supplement the analysis of section (Mbbg). Referring to the periodic control volume indicated in figure 1, and applying the momentum theorem to this control volume, the forces, F_x and F_y, imposed by the fluid on each blade (per unit depth normal to the sketch), are given by

$$F_x = -(p_2 - p_1)h \tag{Mbcb1}$$

$$F_y = \rho h v_m (w_1 \cos\beta_1 - w_2 \cos\beta_2) \tag{Mbcb2}$$

where, as a result of continuity, $v_{m1} = v_{m2} = v_m$. Note that F_y is entirely consistent with the expression (Mbbg5) for the torque, T.

To proceed, we define the vector mean of the relative velocities, w_1 and w_2, as having a magnitude w_M and a direction β_M, where by simple geometry

$$\cot\beta_M = \frac{1}{2}\left(\cot\beta_1 + \cot\beta_2\right) \tag{Mbcb3}$$

$$w_M = v_m \big/ \sin\beta_M \tag{Mbcb4}$$

It is conventional and appropriate (as discussed below) to define the lift, L, and the drag, D, components of the total force on a blade, $(F_x^2 + F_y^2)^{\frac{1}{2}}$, as the components normal and tangential to the vector mean velocity, w_M. More specifically, as shown in figure 1,

$$L = -F_x \cos\beta_M + F_y \sin\beta_M \tag{Mbcb5}$$

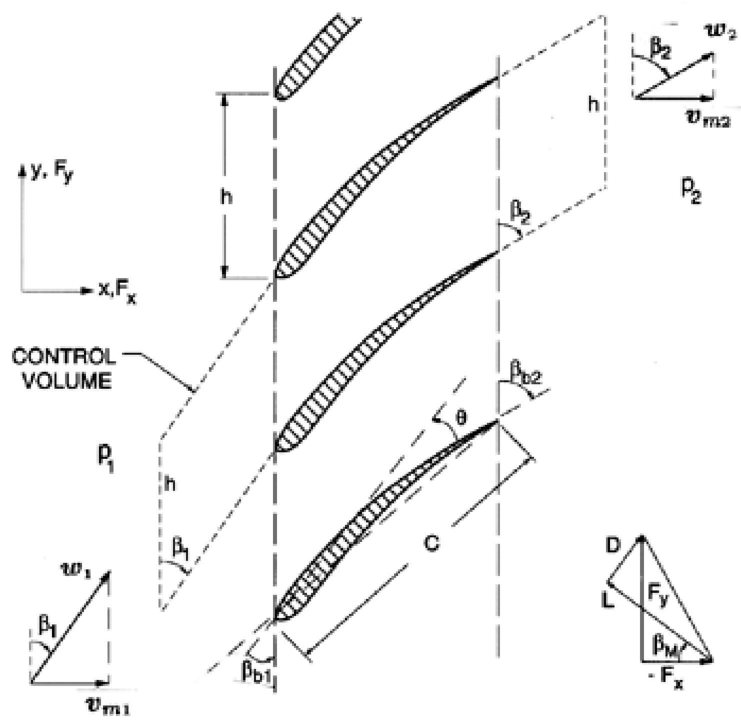

Figure 1: Schematic of a linear cascade showing the blade geometry, the periodic control volume and the definition of the lift, L, and drag, D, forces on a blade.

$$D = F_x \sin \beta_M + F_y \cos \beta_M \tag{Mbcb6}$$

where L and D are forces per unit depth normal to the sketch. Nondimensional lift and drag coefficients are defined as

$$C_L = L/\frac{1}{2}\rho w_M^2 c \; ; \; C_D = D/\frac{1}{2}\rho w_M^2 c \tag{Mbcb7}$$

The list of fundamental relations is complete if we write the expression for the pressure difference across the cascade as

$$p_1 - p_2 = \Delta p_L^T - \frac{\rho}{2}\left(w_1^2 - w_2^2\right) \tag{Mbcb8}$$

where Δp_L^T denotes the total pressure loss across the cascade caused by viscous effects. In frictionless flow, $\Delta p_L^T = 0$, and the relation (Mbcb8) becomes the Bernoulli equation in rotating coordinates (equation (Mbbg1) with $r_1 = r_2$ as is appropriate here). A nondimensional loss coefficient, f, is defined as

$$f = \Delta p_L^T / \rho w_M^2 \tag{Mbcb9}$$

Equations (Mbcb1) through (Mbcb9) can be manipulated to obtain expressions for the lift and drag coefficients as follows

$$C_D = 2f \sin \beta_M / s \tag{Mbcb10}$$

$$C_L = \frac{2}{s}\left[\frac{\psi}{\phi}\sin \beta_M + \frac{f(\phi - \cos \beta_M \sin \beta_M)}{\sin \beta_M}\right] \tag{Mbcb11}$$

where $s = c/h$ is the solidity, ψ is the head coefficient, $(p_2^T - p_1^T)/\rho \Omega^2 R^2$, and ϕ is the flow coefficient, $v_m/\Omega R$. Note that in frictionless flow $C_D = 0$ and $C_L = 2\psi \sin \beta_M (1 - 2\sin^2 \beta_M)/\phi s$; then the total force (lift) on the foil is perpendicular to the direction defined by the β_M of equation (Mbcb3). This provides confirmation that the directions we chose in defining L and D (see figure 1) were appropriate for, in frictionless flow, C_D must indeed be zero.

Also note that equations (Mbcb1) through (Mbcb9) yield the head/flow characteristic given by

$$\psi = \phi\left(\cot \beta_1 - \cot \beta_2\right) - \frac{f\phi^2}{\sin^2 \beta_M} \tag{Mbcb12}$$

which, when there is no inlet swirl or prerotation so that $\tan \beta_1 = \phi$, becomes

$$\psi = 1 - \phi \cot \beta_2 - f\left[\phi^2 + \frac{1}{4}\left(1 + \phi \cot \beta_2\right)^2\right] \tag{Mbcb13}$$

In frictionless flow, when the discharge is parallel with the blades ($\beta_2 = \beta_{b2}$), this, of course, reduces to the characteristic equation (Mbbg4).

Note also that the use of the relation (Mbcb13) allows us to write the expression (Mbcb11) for the lift coefficient as

$$C_L = \frac{2}{s}\left[2\sin \beta_M (\cot \beta_1 - \cot \beta_M) - f \cos \beta_M\right] \tag{Mbcb14}$$

Figure 2 presents examples of typical head/flow characteristics resulting from equation (Mbcb13) for some chosen values of β_2 and the friction coefficient, f. It should be noted that, in any real turbomachine, f will not be constant but will vary substantially with the flow coefficient, ϕ, which determines the angle of incidence and other flow characteristics. More realistic cases are presented a little later in figure 4.

The observant reader will have noted that all of the preceding equations of this section involve only the inclinations of the flow and *not* of the blades, which have existed only as ill-defined objects that achieve

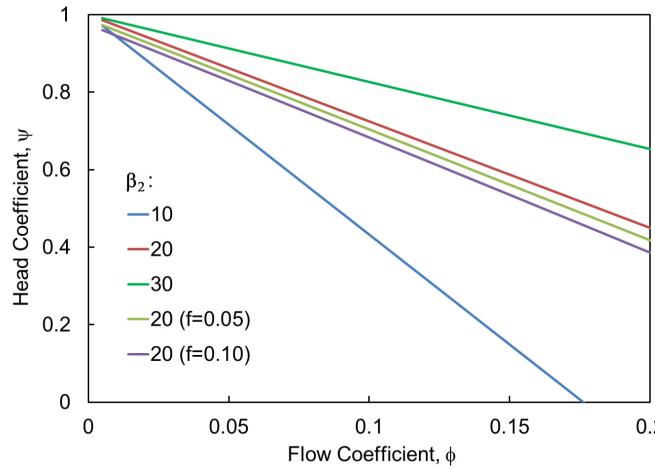

Figure 2: Calculated head/flow characteristics for some linear cascades.

the turning of the flow. In order to progress further, it is necessary to obtain a detailed solution of the flow, one result of which will be the connection between the flow angles (β_M, β_2) and the geometry of the blades, including the blade angles (β_b, β_{b1}, β_{b2}). A large literature exists describing methods for the solutions of these flows, but such detail is beyond the scope of this text. As in most high Reynolds number flows, one begins with potential flow solutions, for which the reader should consult a modern text, such as that by Horlock (1973), or the valuable review by Roudebush (1965). König (1922) produced one of the earliest potential flow solutions, namely that for a simple flat plate cascade of infinitely thin blades. This was used to generate figure 3. Such potential flow methods must be supplemented by viscous analyses of the boundary layers on the blades and the associated wakes in the discharge flow. Leiblein (1965) provided an excellent review of these viscous flow methods, and some of his basic methodology will be introduced later.

To begin with, however, one can obtain some useful insights by employing our basic knowledge and understanding of lift and drag coefficients obtained from tests, both those on single blades (airfoils, hydrofoils) and those on cascades of blades. One such observation is that the lift coefficient, C_L, is proportional to the sine of the angle of attack, where the angle of attack is defined as the angle between the mean flow direction, β_M, and a mean blade angle, β_{bM}. Thus

$$C_L = m_L \sin(\beta_{bM} - \beta_M) \tag{Mbcb15}$$

where m_L is a constant, a property of the blade or cascade geometry. In the case of frictionless flow ($f = 0$), the expression (Mbcb15) may be substituted into equation (Mbcb14), resulting in an expression for β_M. When this is used with equation (Mbcb13), the following head/flow characteristic results:

$$\psi = \frac{2m_L s \sin\beta_{bM}}{4 + m_L s \sin\beta_{bM}}\left[1 - \phi\left(\cot\beta_{bM} + \frac{v_{\theta 1}}{v_{m1}}\right)\right] \tag{Mbcb16}$$

where, for convenience, the first factor on the right-hand side is denoted by

$$\psi_0 = \frac{2m_L s \sin\beta_{bM}}{4 + m_L s \sin\beta_{bM}} = \left[1 + \frac{\cot\beta_2 - \cot\beta_{b2}}{\cot\beta_1 - \cot\beta_2}\right]^{-1} \tag{Mbcb17}$$

The factor, ψ_0, is known as the frictionless shut-off head coefficient, since it is equal to the head coefficient at zero flow rate. The second expression for ψ_0 follows from the preceding equations, and will be used later. Note that, unlike equation (Mbcb13), the head/flow characteristic of equation (Mbcb16) is given

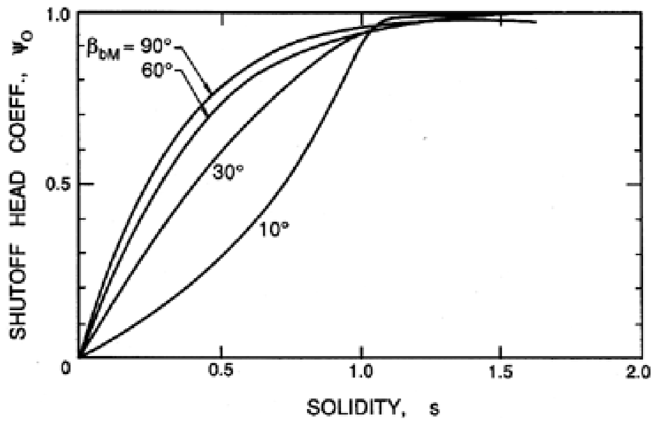

Figure 3: The performance parameter, ψ_0, as a function of solidity, s, for flat plate cascades with different blade angles, β_b. Adapted by Wislicensus (1947) (see also Sabersky, Acosta and Hauptmann 1989) from the potential flow theory of König (1922).

in terms of m_L and practical quantities, such as the blade angle, β_{bM}, and the inlet swirl or prerotation, $v_{\theta 1}/v_{m1}$.

It is also useful to consider the drag coefficient, C_D, for it clearly defines f and the viscous losses in the cascade. Instead of being linear with angle of attack, C_D will be an even function so an appropriate empirical result corresponding to equation (Mbcb15) would be

$$C_D = C_{D0} + m_D \sin^2(\beta_{bM} - \beta_M) \tag{Mbcb18}$$

where C_{D0} and m_D are constants. Some head/flow characteristics resulting from typical values of C_{D0} and m_D are shown in figure 4. Note that these performance curves have a shape that is closer to practical performance curves than the constant friction factor results of figure 2.

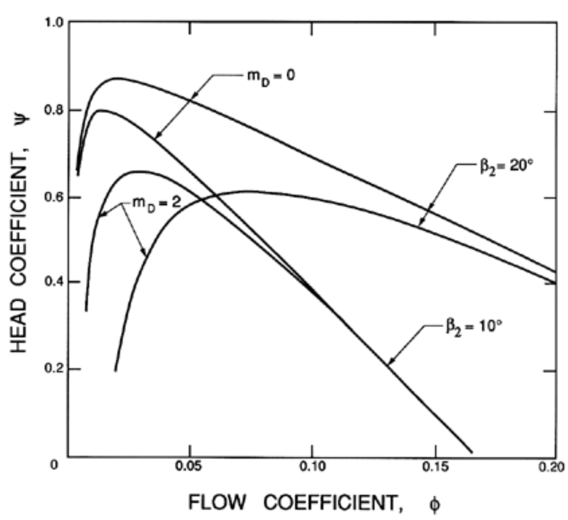

Figure 4: Calculated head/flow characteristics for a linear cascade using blade drag coefficients given by equation (Mbcb18) with $C_{D0} = 0.02$. The corresponding characteristics with $C_{D0} = m_D = 0$ are shown in figure 2.