Flow Notation

The flow variables that are important are, of course, the static pressure, p, the total pressure, p^T, and the volume flow rate, Q. Often the total pressure is defined by the total head, $p^T/\rho g$. Moreover, in most situations of interest in the context of turbomachinery, the potential energy associated with the earth's gravitational field is negligible relative to the kinetic energy of the flow, so that, by definition

$$p^T = p + \frac{1}{2}\rho v^2 \qquad \text{(Mbbc1)}$$

$$p^T = p + \frac{1}{2}\rho \left(v_m^2 + v_\theta^2\right) \qquad \text{(Mbbc2)}$$

$$p^T = p + \frac{1}{2}\rho \left(w^2 + 2r\Omega v_\theta - \Omega^2 r^2\right) \qquad \text{(Mbbc3)}$$

using the velocity triangle of figure 1. In an incompressible flow, the total pressure represents the total

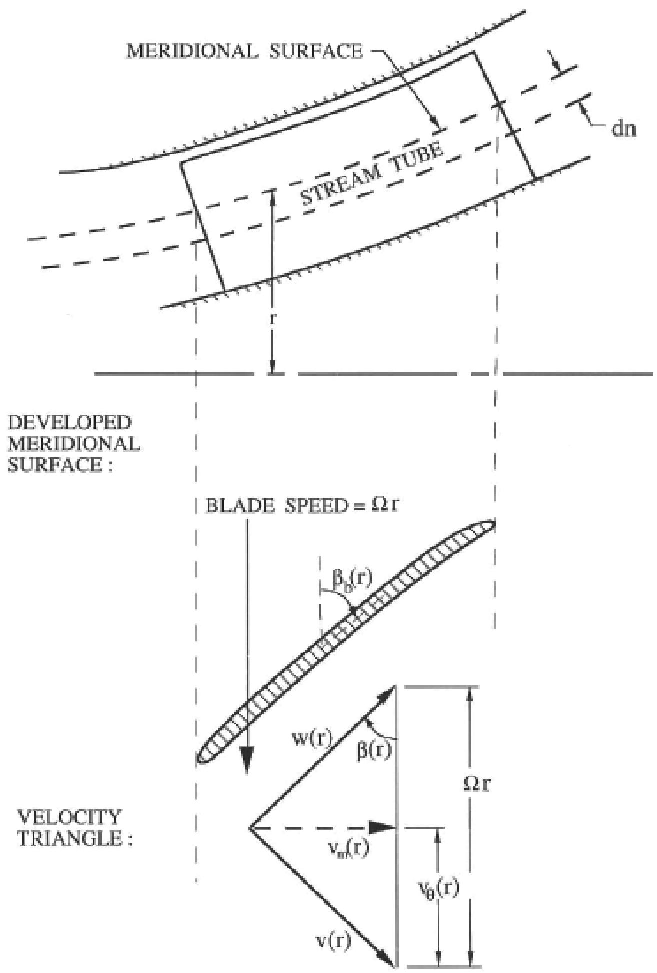

Figure 1: Developed meridional surface and velocity triangle.

mechanical energy per unit volume of fluid, and, therefore, the change in total pressure across the pump, $p_2^T - p_1^T$, is a fundamental measure of the mechanical energy imparted to the fluid by the pump.

It follows that, in a pump with an incompressible fluid, the overall quantities that are important (in addition to the rotation rate, Ω in radians/sec) are the volume flow rate, Q, and the total pressure rise, $\rho g H$, where $H = (p_2^T - p_1^T)/\rho g$ is the total head rise. These dimensional characteristics are conveniently nondimensionalized by defining a head coefficient, ψ,

$$\psi \quad = \quad (p_2^T - p_1^T)/\rho R_{T2}^2 \Omega^2 \quad = \quad gH/R_{T2}^2 \Omega^2 \tag{Mbbc4}$$

and one of two alternative flow coefficients, ϕ_1 and ϕ_2:

$$\phi_1 = Q/A_1 R_{T1} \Omega \qquad \text{or} \qquad \phi_2 = Q/A_2 R_{T2} \Omega \tag{Mbbc5}$$

where A_1 and A_2 are the inlet and discharge areas, respectively. It is also useful to define a torque coefficient, $\mathcal{T} = T/\rho R_{T1}^5 \Omega^2$ where T is the torque applied by the impeller to the fluid. This is also a power coefficient, $P/\rho R_{T1}^5 \Omega^3$, since the power, $P = T\Omega$.

Figure 2 presents a typical example of a set of dimensionless pump characteristics presented as ratios of the head and power coefficients, ψ and $\mathcal{T}$, to the values of those coefficients at the design point (denoted by the subscript D). The efficiency, η, is also shown. Note that the efficiency is a maximum at the design

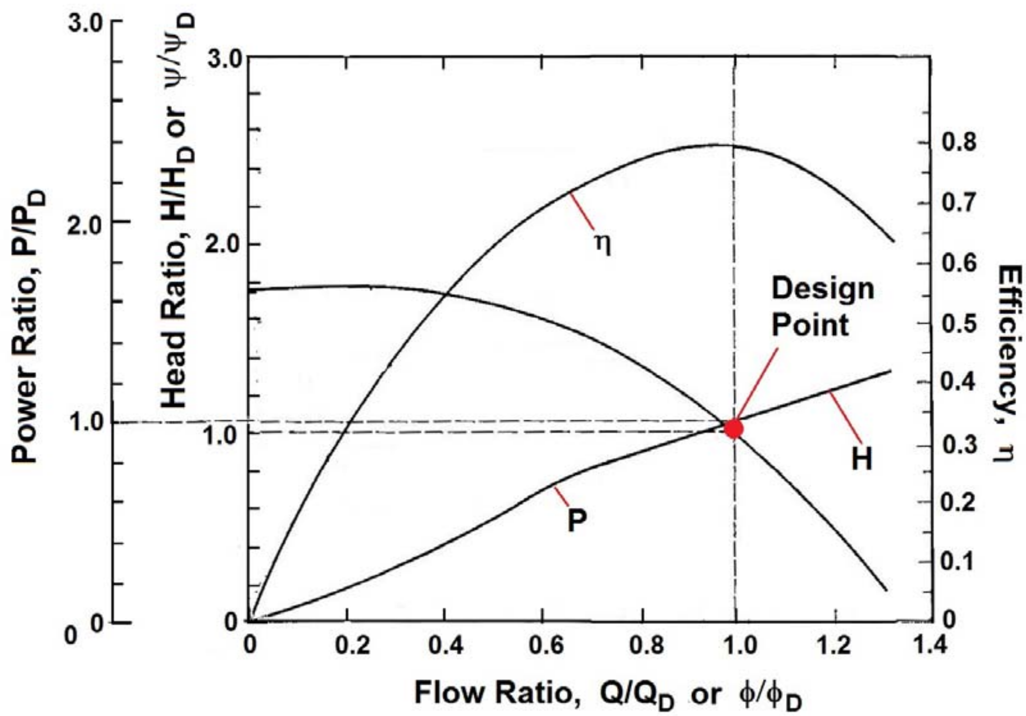

Figure 2: An example of a set of dimensionless pump characteristics, in the case of a $R_{T2} = 0.178m$ centrifugal pump with a specific speed, $N_D = 0.8$, and a discharge blade angle, $\beta_{b2} = 23/circ$. Adapted from Karassik *et al* (1986).

point.

Also note that the discharge flow coefficient is the nondimensional parameter most often used to describe the flow rate. However, in discussions of cavitation, which occurs at the inlet to a pump impeller, the inlet flow coefficient is a more sensible parameter. Note that, for a purely axial inflow, the incidence angle is determined by the flow coefficient, ϕ_1:

$$\alpha(r) = \beta_{b1}(r) - \tan^{-1}(\phi_1 r/R_{T1}) \tag{Mbbc6}$$

Furthermore, for a given deviation angle, specifying ϕ_2 fixes the geometry of the velocity triangle at discharge from the pump.

Frequently, the conditions at inlet and/or discharge are nonuniform and one must subdivide the flow into annular streamtubes, as indicated in figure 1. Each streamtube must then be analysed separately, using the blade geometry pertinent at that radius. The mass flow rate, m, through an individual streamtube is given by

$$m = 2\pi \rho r v_m dn \tag{Mbbc7}$$

where n is a coordinate measured normal to the meridional surface, and, in the present text, will be useful in describing the discharge geometry.

Conservation of mass requires that m have the same value at inlet and discharge. This yields a relation between the inlet and discharge meridional velocities, that involves the cross-sectional areas of the streamtube at these two locations. The total volume flow rate through the turbomachine, Q, is then related to the velocity distribution at any location by the integral

$$Q = \int 2\pi r v_m(r) dn \tag{Mbbc8}$$

The total head rise across the machine, H, is given by the integral of the total rate of work done on the flow divided by the total mass flow rate:

$$H = \frac{1}{Q} \int \frac{(p_2^T(r) - p_1^T(r))}{\rho g} 2\pi r v_m(r) dn \tag{Mbbc9}$$

These integral expressions for the flow rate and head rise will be used in later chapters.