Unsteady Propeller Flow

In this section we examine the unsteady flow through a propeller in a water tunnel. To include the unsteady flow contributions it is necessary to revisit and revise the basic conservation results presented in Section (Mfc). This is a necessary prerequisite to understanding global instabilities of propeller flows such as the surge instability observed and documented by Duttweiler and Brennen (2002). Another potential application is to the sometimes severe structural vibration that can arise due to the interaction between a ship's propeller and the wake of the hull. The body of work on propeller-hull interactions has been summarized by Weitendorf (1989).

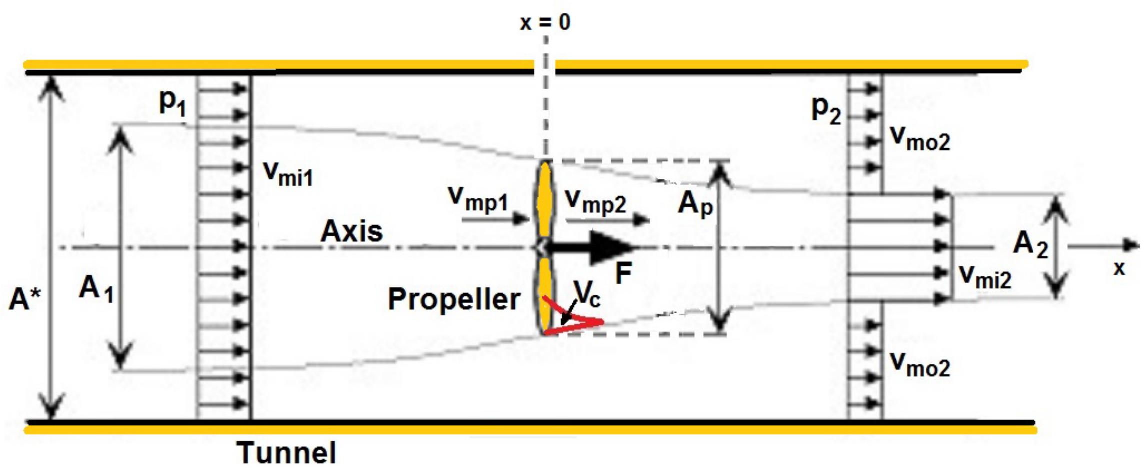

Figure 1: Schematic and notation of a propeller in a tunnel (the cavitation volume is shown in red).

Consider the one-dimensional unsteady, incompressible flow through a propeller (either cavitating or non-cavitating) in a water tunnel as shown in Figure 1. The propeller (cross-sectional area A_p) is located on the centerline of the tunnel whose cross-sectional area is A^*. We focus on the stream tube containing the propeller. For simplicity, it will be assumed that the flow in the propeller streamtube is one-dimensional and uniformly distributed within that stream tube. Friction and mixing losses between the inner and outer flows are neglected.

The analysis parallels that for steady flow detailed in sections (Mfc) and (Mfg) and the notation is the same as that used in those sections. Mass conservation requires that

$$v_{mi1}A_1 - v_{mp1}A_p = -\int_{-\infty}^{0} \frac{\partial A(x,t)}{\partial t}\, dx \tag{Mfi1}$$

$$v_{mi2}A_2 - v_{mp2}A_p = \int_{0}^{\infty} \frac{\partial A(x,t)}{\partial t}\, dx \tag{Mfi2}$$

$$v_{mp2}A_p - v_{mp1}A_p = \frac{dV_c}{dt}\, dt \tag{Mfi3}$$

$$v_{mi2}A_2 + v_{mo2}(A^* - A_2) - v_{mi1}A^* = \frac{dV_c}{dt}\, dt \tag{Mfi4}$$

The right-hand-sides of equations (Mfi1) and (Mfi2) represent the volume change of the stream tube upstream and downstream of the propeller; later these will be ignored for simplicity. The relation between the pressures far upstream and far downstream is obtained by applying Bernoulli's equation in the outer flow as follows:

$$p_2 - p_2 = \frac{1}{2}\rho\left\{v_{mi1}^2 - v_{mo2}^2\right\} - \rho\int_{-\infty}^{\infty}\frac{\partial v_{mo}(x,t)}{\partial t}\,dx \tag{Mfi5}$$

where the last term of the right-hand-side is the inertia effect in the control volume.

Now, we calculate the thrust force F produced by the propeller by applying three basic equations. First, applying the momentum theorem to a control volume containing all the tunnel flow, we obtain;

$$\rho v_{mi1}^2 A^* + p_1 A^* + F \;=\; \rho v_{mo2}^2 (A^* - A_2) + \rho v_{mi2}^2 A_2 + p_2 A^* + \frac{dM}{dt} \tag{Mfi6}$$

The last term in the right-hand-side is rate of the change of the momentum in the control volume, represented by

$$\begin{aligned}\frac{dM}{dt} &= \rho\frac{d}{dt}\left[\int_{-\infty}^{\infty}\left\{v_{mi}(x,t)A(x,t) + v_{mo}(x,t)(A^* - A(x,t))\right\}dx\right] \\ &= \rho\frac{d}{dt}\left[\int_0^{\infty}\frac{dV_c}{dt}\,dx + A^*\int_{-\infty}^{\infty} v_{mi1}dx\right] \;=\; \rho\int_0^{\infty}\frac{d^2V_c}{dt^2}\,dx + \rho A^*\int_{-\infty}^{\infty}\frac{dv_{mi1}}{dt}\,dx\end{aligned} \tag{Mfi7}$$

which yields

$$\begin{aligned}F &= \frac{1}{2}\rho(v_{mi1} - v_{mo2})A^*(2v_{mi2} + v_{mo2} - v_{mi1}) + \rho(v_{mi2} + v_{mo2})\frac{dV_c}{dt} \\ &\quad + \left[\rho A^*\int_{-\infty}^{\infty}\frac{\partial(v_{mi1} - v_{mo}(x,t))}{\partial t}\,dx + \rho\int_0^{\infty}\frac{d^2V_c}{dt^2}\,dx\right]\end{aligned} \tag{Mfi8}$$

Second, we obtain the total pressure difference across the propeller, Δp^T, from the Euler head,

$$\Delta p^T \;=\; \rho R\Omega v_{\theta p2} \;=\; \rho R\Omega(R\Omega - v_{mp2}\cot\beta) - \rho\frac{c}{\sin\beta}\frac{dv_{mp2}}{dt} \tag{Mfi9}$$

The last term in this equation represents the inertia effect of the fluid in the blade passage. Since the static pressure difference, $p_{p2} - p_{p1}$, is given by

$$p_{p2} - p_{p1} \;=\; \frac{1}{2}\rho\left\{R^2\Omega^2 - v_{mp2}^2\cot\beta\right\} - \rho\frac{c}{\sin\beta}\frac{dv_{mp2}}{dt} \tag{Mfi10}$$

the thrust force can be computed as

$$\begin{aligned}F &= (p_{p2} - p_{p1})A_p + \rho\left\{v_{mp2}^2 - v_{mp1}^2\right\}A_p \\ &= \frac{1}{2}\rho\left\{R^2\Omega^2 - v_{mp2}^2\cot\beta\right\}A_p + \rho(v_{mp2} + v_{mp1})\frac{dV_c}{dt} - \rho\frac{A_p c}{\sin\beta}\frac{dv_{mp2}}{dt}\end{aligned} \tag{Mfi11}$$

Third, the pressures p_{p1} and p_{p2} may be related to the upstream and downstream conditions using Bernoulli's equation:

$$p_{p1} \;=\; p_1 + \frac{1}{2}\rho v_{mi1}^2 - \frac{1}{2}\rho v_{mp1}^2 - \rho\int_{-\infty}^{0}\frac{\partial v_{mi}(x,t)}{\partial t}\,dx \tag{Mfi12}$$

where the last term is the inertance in the stream tube. Applying Bernoulli's equation between the outlet of the propeller and far downstream, we obtain

$$p_{p2} \;=\; p_2 + \frac{1}{2}\rho\left[v_{mi2}^2 + v_{\theta p2}^2(A_p/A_2)\right] - \frac{1}{2}\rho\left[v_{mp2}^2 + v_{\theta p2}^2\right] + \rho\int_0^{\infty}\frac{\partial v_{mi}(x,t)}{\partial t}\,dx$$

$$= p_2 + \frac{1}{2}\rho v_{mi2}^2 - \frac{1}{2}\rho v_{mp2}^2 + \frac{1}{2}\rho \left[R\Omega - v_{mp2}\cot\beta\right]^2 \left[(A_p/A_2) - 1\right] + \rho \int_0^\infty \frac{\partial v_{mi}(x,t)}{\partial t}\, dx \quad \text{(Mfi13)}$$

Then the thrust force F follows as

$$\begin{aligned} F &= (p_{p2} - p_{p1})A_p + \rho\left\{v_{mp2}^2 - v_{mp1}^2\right\} A_p \\ &= \frac{1}{2}\rho\left[\left\{v_{mi2}^2 - v_{mo2}^2\right\} + \left\{R\Omega - v_{mp2}\cot\beta\right\}^2 \left\{(A_p/A_2) - 1\right\}\right] A_p \\ &\quad -\frac{1}{2}\rho(v_{mp2} + v_{mp1})\frac{dV_c}{dt} + \rho A_p \int_0^\infty \frac{\partial(v_{mi}(x,t) - v_{mo}(x,t))}{\partial t}\, dx \end{aligned} \quad \text{(Mfi14)}$$

For the purpose of the general discussion, we have considered all possible unsteady effects in the above

formulation, namely the effects of volume change of the stream tubes in equations (Mfi1) and (Mfi2), the inertia effects upstream and downstream of the propeller in equations (Mfi5), (Mfi8) and (Mfi14), and the inertia effect in the propeller in equation (Mfi11) as well as the effects of the cavity volume change dV_c/dt in equations (Mfi3) and (Mfi4). To evaluate many of these terms, it is necessary to know the shape of the stream tube, which is beyond the scope of the present treatment. Some compromises could be implemented in order to simplify the analysis. First, the stream tube volume changes in equations (Mfi1) and (Mfi2) could be neglected on the basis that these cancel and thus produce no net perturbation within the propeller installation. Second, based on other experiences, the inertance terms in equations (Mfi5), (Mfi8) and (Mfi14) could be lumped into the inertance contributions from the flows upstream and downstream of the propeller. On the other hand the unsteady effects caused by cavitation and associated with dV_c/dt in equations (Mfi3) and (Mfi4) can have important consequences and require further development.

Summarizing, the eight equations (Mfi1) through (Mfi14) represent the a dynamic model of the unsteady flow through a propeller. The equations contain eight unknowns v_{mo2}, v_{mi2}, v_{mp2}, v_{mp1}, A_1, A_2, F, and p_2 assuming that the propeller operating parameters v_{mi1}, p_1, $R\Omega$, the discharge flow angle, β, and the rate of change of the cavity volume, dV_c/dt, are given. The empirical relations for the deviation angle that were described in section (Mfg) could be used to supplement the model.

To complete the set of governing equations a functional expression for the cavity volume, V_c, is required when cavitation is involved. Consistent with the understanding developed in the context of cavitating pumps (Section Nrq) it is assumed that, at low frequencies at which a quasistatic approach is appropriate, the cavity volume, $V_c(p_{p1}, v_{mp1})$, is a function of the inlet pressure p_{p1} and inflow velocity v_{mp1}. Then, the rate of change of the cavity volume can be expressed as

$$\frac{dV_c}{dt} = -K\frac{dp_{p1}}{dt} - M\frac{dv_{mp1}}{dt} \quad \text{(Mfi15)}$$

where $K = -\partial V_c/\partial p_{p1}$ and $M = -\partial V_c/\partial v_{mp1}$ are respectively the cavitation compliance and the mass flow gain factor (see Section (Nrq) and Brennen and Acosta 1973). These important parameters are non-dimensionalized as follows;

$$\frac{K^*}{2\pi} = -\frac{\partial(V_c/A_pR)}{\partial\sigma^*} = \frac{\rho R\Omega^2}{2A_p}\frac{\partial V_c}{\partial p_{p1}} = \frac{\rho\Omega^2}{2\pi R}K \quad \text{(Mfi16)}$$

$$M^* = -\frac{\partial(V_c/A_pR)}{\partial(v_{mp1}/R\Omega)} = \frac{\Omega}{A_p}\frac{\partial V_c}{\partial v_{mp1}} = \frac{\Omega}{\pi R^2}M \quad \text{(Mfi17)}$$

where K^* and M^* are the non-dimensional values of the cavitation compliance and the mass flow gain factor (Duttweiler and Brennen 2002). Analyses of the above system of equations demonstrate that K^*

and M^* are functions of the dimensionless mean flow parameters, the advance ratio, J_1, (or flow coefficient, ϕ) and the cavitation number, σ and the reduced frequency, ω/Ω.

Estimated values for $K^*/2\pi$ and M^* for a cavitating propeller have been proposed by Otsuka *et al.* (1996) and Watanabe *et al.* (1998). They examined the unsteady planar flow through a cavitating cascade using free streamline methods (see Section (Nui)) and evaluated the cavity size per blade as a cross-sectional area V_{cpb} (not a volume). This was then applied to the three-dimensional propeller flow using the crude estimate $V_c = Z_R R V_{cpb}/2$. It transpired that the resulting values of $K^*/2\pi$ and M^* are primarily functions of the parameter $\lambda = \sigma^*/2\alpha$, where σ^* is the cavitation number *at inlet to the propeller*. Those values of $K^*/2\pi$ and M^* are shown in Figure 2 for a propeller with typical values for the solidity (1.0), stagger angle ($\beta = 25°$) and number of blades ($Z_R = 5$).

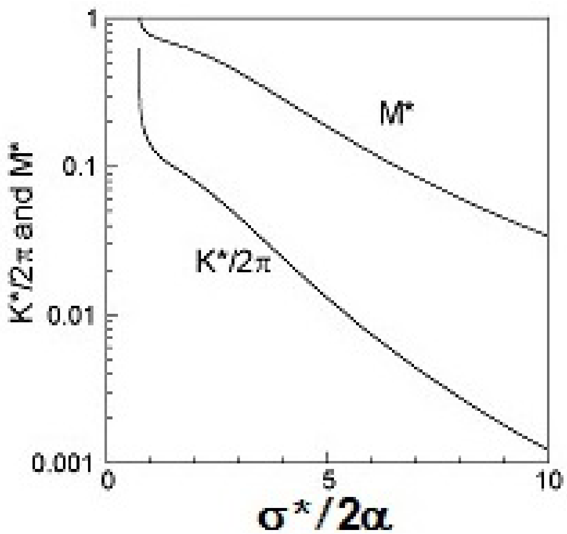

Figure 2: The estimated quasi-static cavitation compliance, $K^*/2\pi$, and mass flow gain factor, M^*, obtained by Watanabe *et al.* (1998) using a free streamline approach. The results plotted against $\sigma^*/2\alpha$ are for a solidity of = 1.0, a stagger angle $\beta = 25°$ and number of blades, $Z_R = 5$.

This completes a model for the low frequency dynamics of a cavitating propeller. In Section (Nrs) typical low frequency dynamic transfer functions that result from this model are presented and analyzed.