# Propulsion Efficiency

In aircraft and rocket design, overall propulsive efficiency, $\eta_t$, is the fraction of the energy contained in a vehicle's propellant that is converted into useful energy used to propel the vehicle. It is represented as $\eta_t = \eta_c\eta_p$ where $\eta_c$ is the cycle efficiency and $\eta_p$ is the propulsive efficiency. The propulsive efficiency, can, in turn, be divided into a shaft efficiency, $\eta_s$, representing the fraction of the energy contained in a vehicle's propellant that is represented in the rotating energy of the shaft and $\eta$, the propeller efficiency, which is the fraction of the shaft energy that is used to propel the vehicle. Herein we focus only on that propeller efficiency, $\eta$, since that is inherent in the design of the propulsive device.

Consider first propulsion effected primarily by drag, that is to say the force produced by dragging an oar or the paddle of a paddle steamer through the water as sketched schematically in Figure 1. The boat is

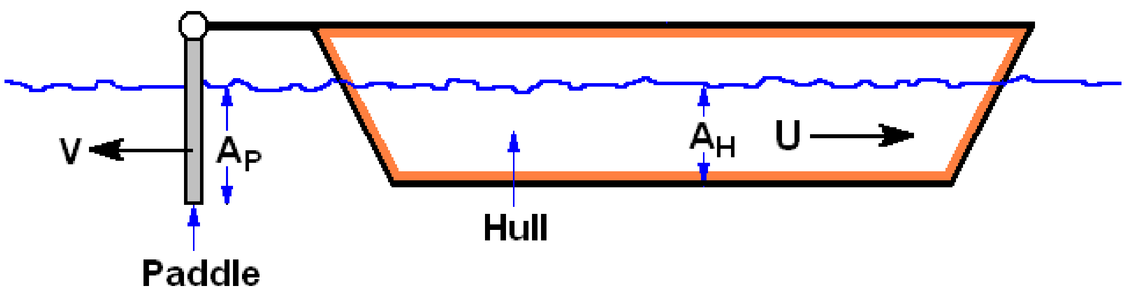


Figure 1: Schematic of a boat propelled by a paddle or oar.

moving forward through the water at a velocity, $U$, propelled by a paddle or oar that is dragged backward through the water with a velocity, $V$, relative to the boat. The effective frontal area of the boat is denoted by $A_H$ while the frontal projected area of the paddle is denoted by $A_P$. Then, if the drag coefficient for the hull is denoted by $C_{DH}$ and that for the paddle is denoted by $C_{DP}$, it follows that since the thrust produced by the paddle must be equal to the drag on the hull, that

$$\frac{1}{2}\rho\, C_{DP}A_P(V-U)^2 \;=\; \frac{1}{2}\rho\, C_{DH}A_HU^2 \qquad \text{(Ddc1)}$$

and therefore

$$\frac{V}{U} \;=\; 1+\left\{\frac{C_{DH}A_H}{C_{DP}A_P}\right\}^{\frac{1}{2}} \qquad \text{(Ddc2)}$$

Since both drag coefficients will be of order unity, it follows that the ratio $V/U$ is primarily determined by the area ratio, $A_H/A_P$ and the larger this ratio the larger $V/U$ must be.

Now examine the efficiency of this method of propulsion. The useful rate of work done is that used in propelling the boat namely

$$\frac{1}{2}\rho\, C_{DH}A_HU^3 \qquad \text{(Ddc3)}$$

while the rate of work done moving the paddle through the water is

$$\frac{1}{2}\rho\, C_{DP}A_P(V-U)^3 \qquad \text{(Ddc4)}$$

so the efficiency, $\eta$, is

$$\eta = \frac{\frac{1}{2}\rho\, C_{DH}A_H U^3}{\frac{1}{2}\rho\, C_{DH}A_H U^3 + \frac{1}{2}\rho\, C_{DP}A_P(V-U)^3} \tag{Ddc5}$$

or

$$\eta = \frac{1}{1+\left\{\frac{V}{U}-1\right\}^3 \frac{C_{DP}A_P}{C_{DH}A_H}} \tag{Ddc6}$$

and substituting for $V/U$ from equation (Ddc2) this yields

$$\eta = \frac{1}{1+\left\{\frac{C_{DH}A_H}{C_{DP}A_P}\right\}^{\frac{1}{2}}} \tag{Ddc7}$$

Since the drag coefficients will roughly cancel each other it follows that the greater the area ratio $A_P/A_H$, the greater the efficiency of propulsion. Consequently, as the development of paddle steamers progressed, the paddles got larger and larger relative to the hull in order to maximize the efficiency of propulsion.

***

Now consider the simplest model of the propulsion of a boat using a propeller (or screw in marine notation). Figure 2 is a diagram depicting the motion of a propeller blade in which we are now viewing the boat from vertically overhead. The velocity of the blade relative to the hull is denoted by $V$ and this velocity is perpendicular to the direction of motion of the boat, $U$. The movement of the screw blade produces the lift, $L$, which propels the boat at its velocity, $U$, and is equal to the drag on the boat. Consequently

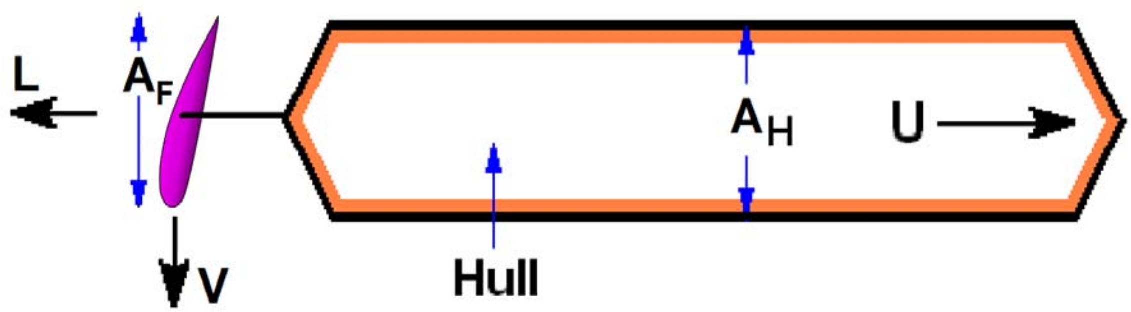


Figure 2: Schematic of a boat propelled by a screw propeller.

$$\frac{1}{2}\rho\, C_{LP}A_P V^2 = \frac{1}{2}\rho\, C_{DH}A_H U^2 \tag{Ddc8}$$

where $C_{LP}$ is the lift coefficient produced by the screw blade and $A_P$ is the effective planform area of all the screw blades. Hence the velocity ratio $V/U$ is given by

$$\frac{V}{U} = \left\{\frac{C_{DH}A_H}{C_{LP}A_P}\right\}^{\frac{1}{2}} \tag{Ddc9}$$

While this expression appears superficially similar to equation (Ddc2) for paddle propulsion, it is important to note that, with a well chosen foil shape and angle of attack for the blade, the value of $C_{LP}$ can be much larger than $C_{DP}$ and consequently, to achieve similar $V/U$ values the typical propeller area $A_P$ can be much smaller than might otherwise be expected.

But the primary advantage of the screw over the paddle is in its propulsive efficiency. The rate of work done to move the screw blades through the water is

$$\frac{1}{2}\rho\, C_{DP} A_P V^3 \tag{Ddc10}$$

where $C_{DP}$ is the drag coefficient for the screw blade. Therefore, the propulsive efficiency is given by

$$\eta \;=\; \frac{\frac{1}{2}\rho\, C_{DH} A_H U^3}{\frac{1}{2}\rho\, C_{DH} A_H U^3 + \frac{1}{2}\rho\, C_{DP} A_P V^3} \;=\; \frac{1}{1 \;+\; \frac{V^3}{U^3}\frac{C_{DP}A_P}{C_{DH}A_H}} \tag{Ddc11}$$

and substituting for $V/U$ from equation (Ddc9) this becomes

$$\eta \;=\; \frac{1}{1 \;+\; \left\{\frac{C_{DH}A_H}{C_{LP}A_P}\right\}^{\frac{1}{2}} \frac{C_{DP}}{C_{LP}}} \tag{Ddc11}$$

Notice that this is very similar to the expression (Ddc9) for the efficiency of paddle propulsion. However, a very important difference is the presence of the drag to lift ratio, $C_{DP}/C_{LP}$, for the screw blade in the denominator of equation (Ddc11). A well-designed screw blade profile and angle of attack can produce a drag to lift ratio much less than unity which will greatly improve propulsive efficiency. Thus a device that uses lift for propulsion (such as a propeller) has the potential for much greater efficiency than one that uses drag for propulsion (such as a paddle). This same property can be seen in many animal propulsion mechanisms and will be detailed in a subsequent section.