Jet Propulsion

The thrust produced by a jet engine is most readily understood through the application of the linear momentum theorem. Consider the sketch of the cross-section of a jet engine as shown in the figure below.

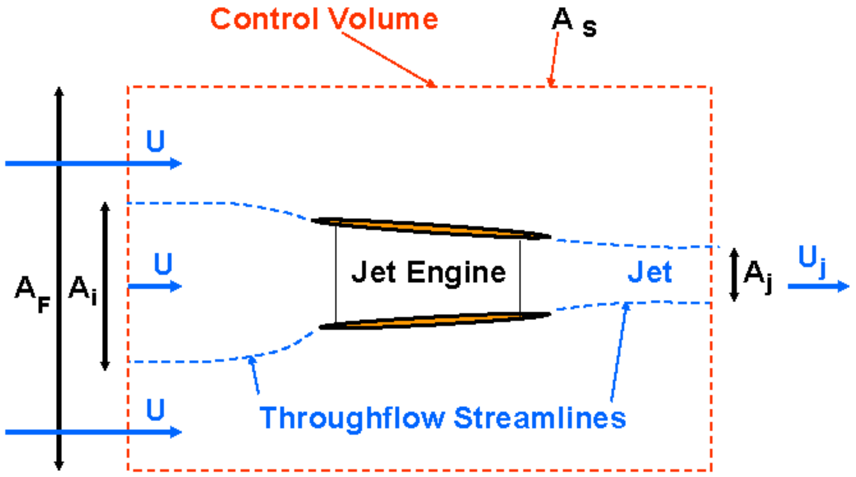

Figure 1: A cross-section through a jet engine showing the outline of the throughflow (dashed blue lines) and a cylindrical control volume (dashed red lines).

The outline of the throughflow is shown by the dashed blue lines. Far upstream the cross-sectional area of this streamtube is denoted by A_i and the velocity within it is the same as the rest of the upstream flow, namely U. Far downstream the cross-section of the jet emerging from the engine is denoted by A_j. The velocity of this jet is denoted by U_j and we assume that any mixing between the jet and the surrounding fluid can be neglected so that the bounding streamtube surface represents a discontinuity in velocity. Viscous effects in the exterior flow are also neglected so that, by Bernoulli's law, the velocity of the fluid exterior to the jet must be U.

We define a large cylindrical control volume as shown by the dashed red lines. The various components of the surface of this control volume are:

- A large upstream area, A_F, normal to the oncoming stream that is sufficiently far from the engine so that the pressure on that surface is essentially the atmospheric pressure far upstream.
- Within A_F, the intersection of the throughflow streamtube with that area is denoted by A_i.
- A large cylindrical surface, A_S, that is everywhere parallel with U and constitutes the outer boundary of the control volume.
- A downstream surface normal to the oncoming stream that is the other end of the cylindrical control volume and therefore also has an area A_F.
- Within this downstream area is the intersection of the jet that has an area A_j.

Therefore, except for the jet area, all the flows on the boundaries of this control volume have a velocity in the U direction equal to U; in contrast, the exiting jet has a velocity U_j which, for simplicity, we will assume is uniform across the jet. Since the pressures on all the boundaries of the control volume are assumed to be equal to the upstream atmospheric pressure it follows that the densities of the entering and exiting flows are as follows. Except for the exiting jet the other entering and exiting flows have the same density as the upstream flow which will be denoted by ρ. In contrast since the temperature of the exiting jet may be much hotter, its density will be denoted by ρ_j.

With these definitions we can now apply conservation of mass and then the momentum theorem in the U direction. The mass rate at which fuel is added to the flow inside the engine is usually very small compared with the throughflow mass flow rate of the air and so we neglect the added fuel mass (the primary effect of the fuel is to add heat by its combustion). Then, assuming that the flow is steady so that the mass of fluid inside the control volume is unchanging then conservation of mass requires that

$$\rho U A = \rho U(A - A_j) + \rho_j U_j A_j + \dot{M} \tag{Dde1}$$

where $\dot{M}$ denotes the mass flow out through the sides of the control volume, namely the area A_S. It follows that

$$\dot{M} = A_j(\rho U - \rho_j U_j) \tag{Dde2}$$

Now we apply the momentum theorem in the U direction to obtain the total force F acting on the contents of the control volume (which includes the jet engine and the jet) in the U direction:

$$F = -\rho U^2 A + \rho U^2(A - A_j) + \rho_j U_j^2 A_j + U\dot{M} \tag{Dde3}$$

and, with cancellations and the substitution for $\dot{M}$ from the expression derived from continuity, this becomes

$$F = \rho_j U_j A_j(U_j - U) \tag{Dde4}$$

Finally we must consider the various possible contributions to the total force, F, acting on the control volume and its contents in the U direction. It is assumed that the flow has a sufficiently high Reynolds number so that the shear stresses acting on A_0 are negligible and so that there are no significant viscous contributions to the normal stresses on the surfaces normal to U. Thus the only pertinent forces acting on the external surface of the control volume are those due to the pressure. Moreover it is assumed that these surfaces are sufficiently far from the body that the pressures on all surfaces are equal to the pressure in the uniform stream. It follows that there is no contribution of the pressures to F. Consequently if we neglect contributions from body forces such as gravity (or assume U is horizontal), the only contribution to F is the force that must be applied to the body to hold it in place. That force will be the thrust produced by the engine, T, defined as the force imposed by the engine on the rest of the airplane (or supporting structure) and acting in the *negative* U direction. It follows that the force, F, on the engine and therefore on the control volume is equal to T in the *positive* U direction. Therefore $F = T$ and

$$T = \rho_j U_j A_j(U_j - U) = \dot{M}_j(U_j - U) \tag{Dde5}$$

Note how the thrust is simply the mass flow through the engine, $\dot{M}_j$, multiplied by the velocity increment, $(U_j - U)$, in the jet.

In this simplified analysis we have assumed that the velocity, U_j, is uniform across the jet area, A_j. Problem 222C explores the effect of a velocity distribution within the jet and shows that such non-uniformity increases the thrust because the regions of higher velocity contribute more momentum flux than is deducted by the regions of low velocity.

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It is appropriate to include a few details of developed jet engines. As depicted in Figure 2, the original, basic jet engine, known as a *turbojet engine*, consists of an air intake followed by an axial compressor with multiple stages. The air is then fed into the combustion chamber where fuel is injected and ignited. The hot gas is then expanded through a gas turbine that drives the compressor. The thrust is produced by the hot gas discharging from the turbine through a subsonic nozzle. Later it was recognized that a more efficient engine would consist of a larger fan (or first stage of the compressor) with some of the discharge from the fan being immediately exhausted. This configuration is known as a *turbofan engine* and is depicted in Figure 3; an example is the RollsRoyceRB211 engine shown in Figure 4. This configuration is not only more efficient but the shroud of cooler air around the turbine exhaust helps with the jet noise. As the design progressed the bypass ratio (fraction of the fan air exhausted to compressor intake) increased to the point at which it was recognized that dispensing with the duct around the fan would be beneficial to the efficiency. Such a design, called a *turbo-prop engine*, is depicted in Figure 5. However, the noise generated at the fan-tip is a substantial defect in this design and has prevented its deployment in passenger aircraft. In a turbo-fan engine the noise generated by the fan tip is substantially absorbed by the acoustic liner built into the interior surface of the engine intake.

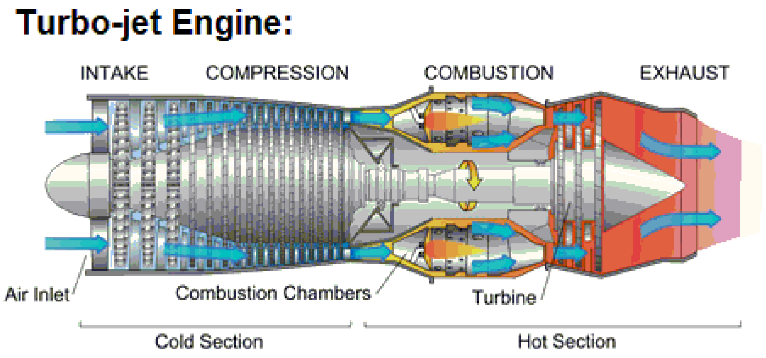

Figure 2: Schematic of a turbo-jet engine.

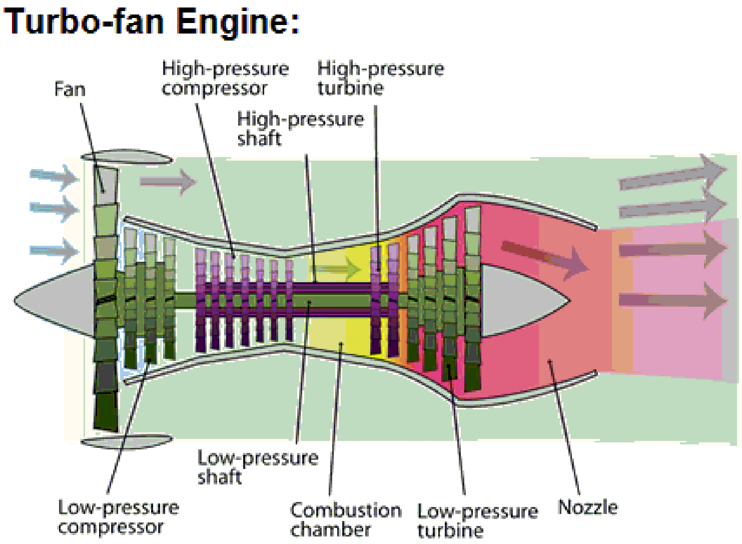

Figure 3: Schematic of a turbo-fan engine.

The focus on reducing aircraft noise has led to considerable decrease in the jet engine noise even as the

Figure 4: The Rolls Royce RB211 turbo-fan engine. Specifications: thrust: $160 - 270kN$, thrust/weight ratio: $4 - 5$, bypass ratio: $5 - 1$, jet temperature/ambient temperature: about 2.5, jet Mach number: about 0.9.

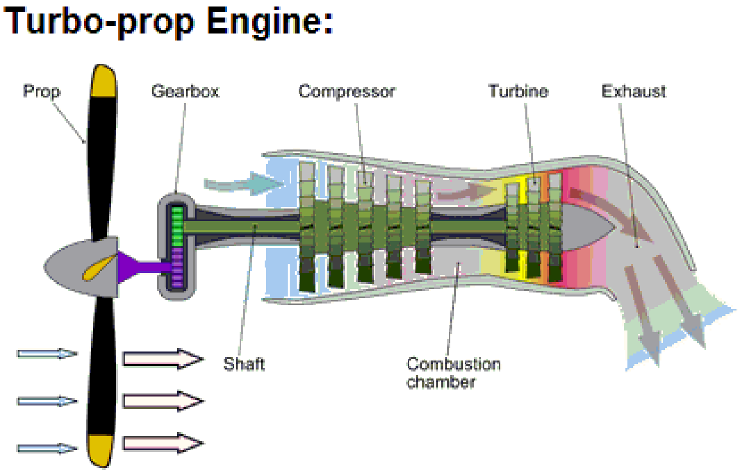

Figure 5: Schematic of a turbo-prop engine.

size of the engines has increased. Figure 6 is a graph of the progress that has been made in this regard since the early days of the jet engine.

Another evolving feature of gas turbines has been the progressive increase in the temperatures and pressures in the combustion chamber in an attempt to improve the cycle efficiency of the engine (note that very similar gas turbines are used for the generation of power and for jet engines). The resulting inlet temperatures to the turbine present serious challenges for the material of the first stage of the turbine. Quite complicated surface cooling techniques have been developed in order to achieve these higher temperatures; one set of these techniques involves injection of water through holes in the blades.

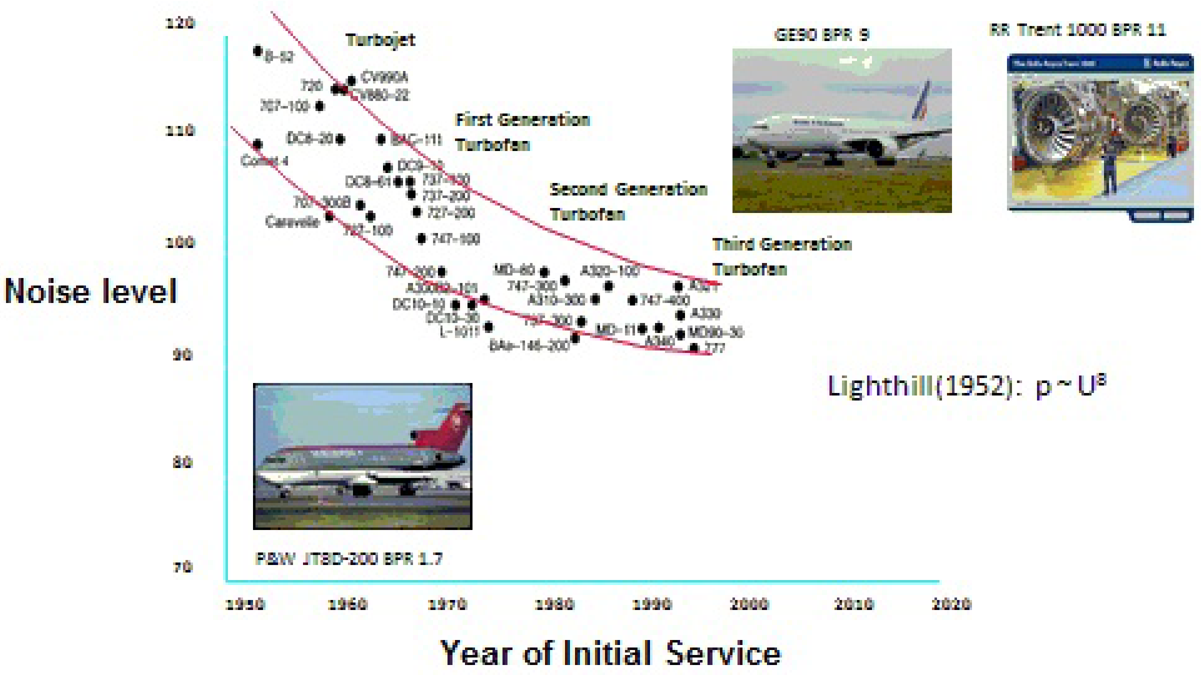

Figure 6: The evolution of jet engine noise since 1950. Data from David Reed, Boeing.