Magnus Effect

The "Magnus Effect" is the lift produced by a rotating cylinder in a uniform stream. It can be predicted using the appropriate potential flow solution for a rotating cylinder in a uniform stream as depicted in Figure 1. In section (Bdgi) we presented the potential flow solution to this flow but we repeat it here for convenience.

The planar incompressible potential flow past a spinning cylinder (radius = R) is constructed by superposition of the velocity potentials for (a) a uniform stream, $\phi = Ux$ (b) a doublet, $\phi = UR^2 \cos\theta / r$ at the center of the cylinder and (c) a potential vortex with circulation, Γ, such that $\phi = \Gamma\theta/2\pi$. Here the circulation, Γ, is defined as positive in the anticlockwise direction. This simulates the flow due to anticlockwise rotation of the cylinder and we will proceed to find the velocity and pressure on the surface of the cylinder as a function of angular position, θ, and subsequently, the lift force, L, acting on the cylinder.

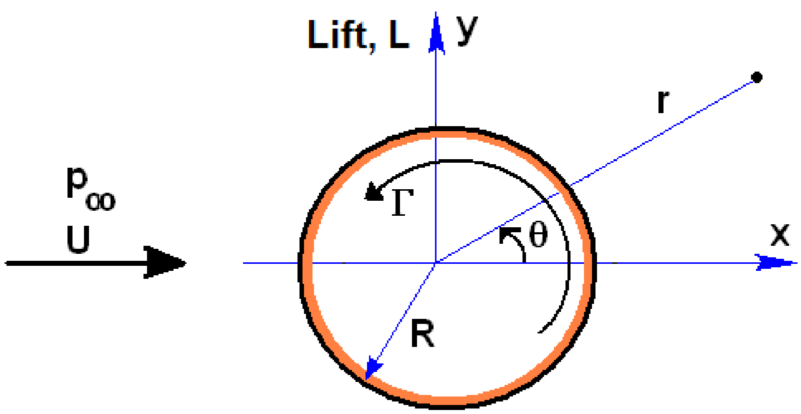

Figure 1: Planar potential flow around a rotating cylinder in a uniform stream.

The incompressible planar potential flow past a spinning cylinder (radius = R) in a uniform stream of velocity, U, in the positive x direction is given by the velocity potential:

$$\phi = Ur\cos\theta + \frac{UR^2\cos\theta}{r} + \frac{\Gamma\theta}{2\pi} \quad \text{(Dcb1)}$$

where r, θ are polar coordinates in which $\theta = 0$ corresponds to the positive x direction and Γ is the circulation defined as positive in the anticlockwise direction. It follows that the velocities, u_r and u_θ in the r and θ directions, are given by

$$u_r = \frac{\partial\phi}{\partial r} = U\cos\theta - \frac{UR^2\cos\theta}{r^2} \quad \text{(Dcb2)}$$

$$u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} = -U\sin\theta - \frac{UR^2\sin\theta}{r^2} + \frac{\Gamma}{2\pi r} \quad \text{(Dcb3)}$$

Consequently the velocities on the surface of the cylinder are

$$(u_r)_{r=R} = 0 \quad \text{(Dcb4)}$$

$$(u_\theta)_{r=R} = -2U\sin\theta + \frac{\Gamma}{2\pi R} \quad \text{(Dcb5)}$$

To find the pressure on the surface, $(p)_{r=R}$, we now use Bernoulli's thereom to obtain

$$(p)_{r=R} = p_\infty + \frac{\rho}{2}\left[U^2(1-4\sin^2\theta) + \frac{2\Gamma U}{\pi R} - \frac{\Gamma^2}{4\pi^2 R^2}\right] \tag{Dcb6}$$

where p_∞ is the pressure in the uniform stream.

Integrating this pressure over the surface in order to obtain the drag, the result for the drag is zero as expected from D'Alembert's paradox.

Integrating the pressure over the surface in order to obtain the lift, L, per unit depth normal to the planar flow,

$$L = \int_0^{2\pi} -(p)_{r=R} R\sin\theta d\theta \tag{Dcb7}$$

which leads to

$$L = -\rho U\Gamma \tag{Dcb8}$$

This is called the Magnus effect. The lift is due to the fact that the cylinder rotation in the anticlockwise direction decreases the surface velocity on the upper side and increases the velocity on the lower side. By Bernoulli's equation this increases the pressure on the upper side and decreases the pressure on the lower side thus causing the negative lift on the cylinder. This effect also occurs with three-dimensional objects such as spheres and is particularly evident in the flight of a well-struck golf ball.