Wave Drag

When an object, such as a ship, is moving through the fluid at a free surface, the production of waves by that ship introduces another dimension to the complexity of the flow and leads to another mechanism of drag. The waves that propagate away from the ship carry with them energy that is directly related to an additional component of drag on the ship, a component known as the *wave drag*. These flows are very complicated as can be illustrated by the typical pattern of waves shown in Figure 1 and sketched in Figure 2. Waves primarily originate at the bow and stern of the ship and radiate away from the ship in two classical patterns that are indicated in Figure 2, namely a set of divergent waves emanating from the bow and stern and a set of related transverse waves. The transverse waves are reflected in the shape of the water surface along the waterline of the ship. This complex wave pattern is primarily confined within the wedge-shaped region indicated in Figure 2; the half-angle of the wedge-shaped region is 19.5^o, named

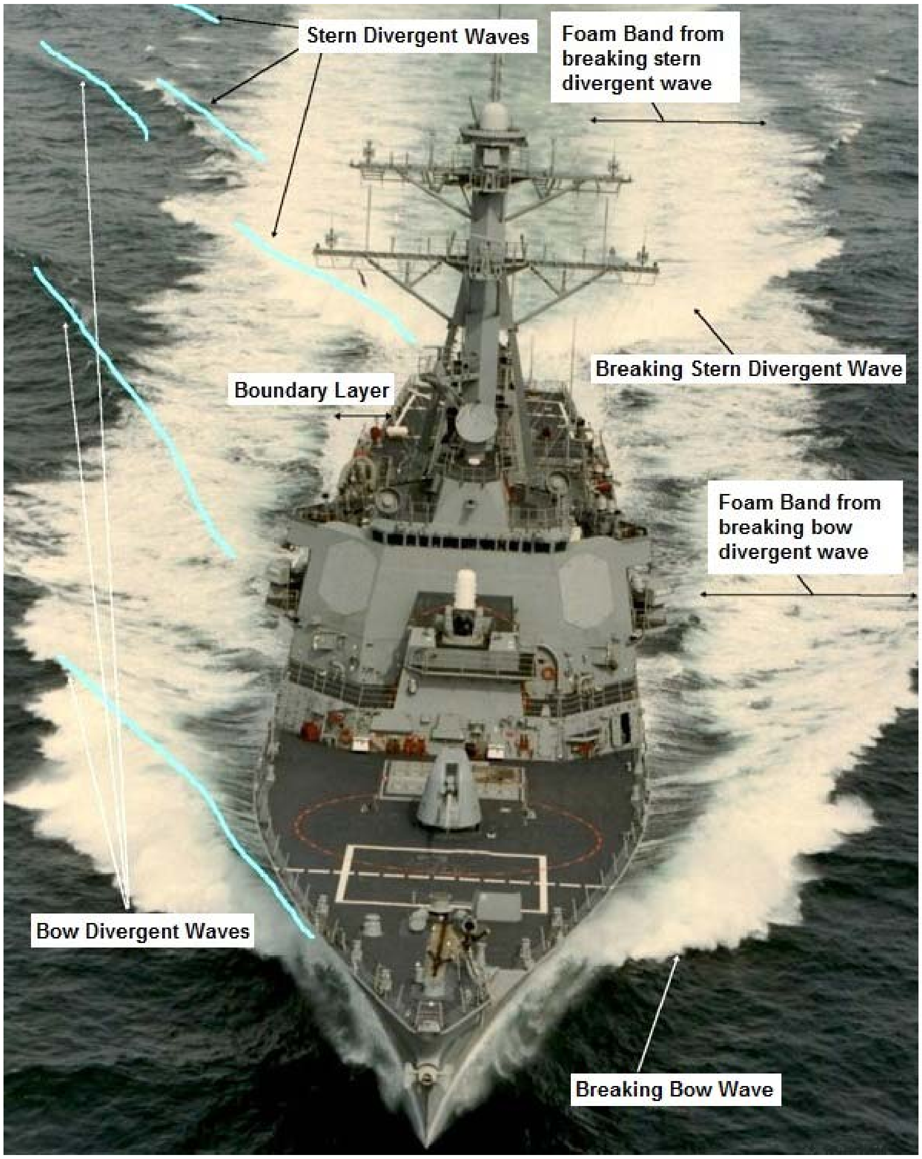

Figure 1: Pattern of waves created by a surface ship.

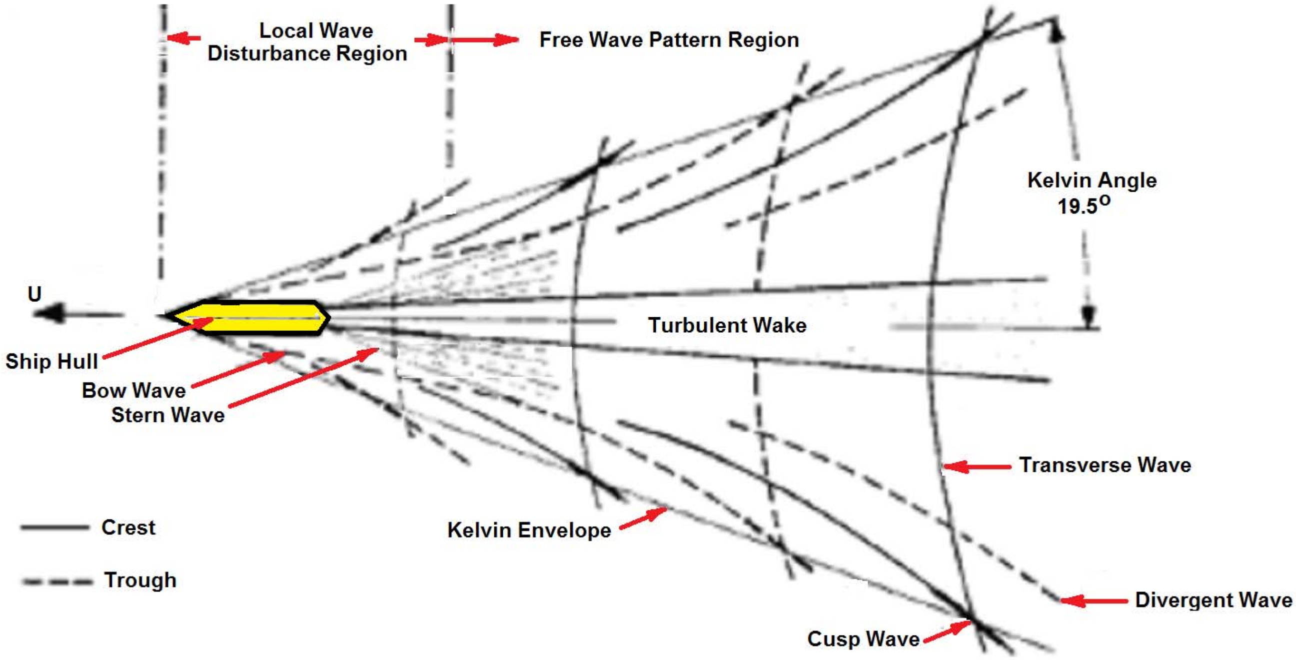

Figure 2: Pattern of waves created by a surface ship with wave notation.

the Kelvin angle after its discoverer.

As illustrated by Figure 1, the large wave produced by the bow of the ship usually breaks making the flow even more complex and essentially impossible to model analytically even by modern numerical methods. Due to the extreme complexity of the flow, evaluation of the additional drag caused by the waves must rely on experimental testing and empirical modeling.

However, it is possible to define one of the parameters that will effect the magnitude of the wave drag coefficient, C_{DW}. Clearly, in the hypothetical circumstances of a very large gravitational acceleration, g, there would be essentially no waves and the surface of the water would be effectively a plane of symmetry in the flow. Such a circumstance would correspond to zero Froude number, $Fr \rightarrow 0$ and there would be no wave drag so that $C_{DW} = 0$. However, in more general circumstances, as described in section (Daa), the drag would be a function of the Froude number, $Fr = U/(g\ell)^{1/2}$ (where ℓ is a typical dimension of the ship, usually taken to be its length), as well as the other parameters listed in section (Daa). Consequently the wave drag coefficient, C_{DW}, is expected to be a predominantly increasing function of Fr. Figure 3 presents some data on measured drag coefficients for a typical ship of various ratios of submerged volume, V, to the cube of the ship length, ℓ^3. Since it very difficult to separately measure the form drag, skin friction drag and wave drag coefficients, the data plotted is the total drag minus the estimated skin friction drag. The plotted drag coefficients are therefore the estimated sum of the form drag and the wave drag but they exhibit some features that are characteristic of the wave drag contribution. We might expect that the small intercept at around 0.0005 on the vertical axis at $Fr = 0.2$ primarily reflects form drag. If so then we would expect the form drag to be about 0.002 at $Fr = 0.4$. Clearly there are additional contributions over and above this which are most likely the contributions from wave drag.

Experiments also show that the wave patterns generated tend to vary significantly depending on the relation between the length of the ship, ℓ, and the wavelength, λ, of the ocean waves. First note that we are primarily interested in waves that travel at the *same speed* as the ship since that is the wave pattern shown in Figures 1 and 2 that is responsible for the wave drag and which is reflected in the profile of the transverse waves along the waterline of the ship. Note also from section (Bgcc) that the speed, c, of waves

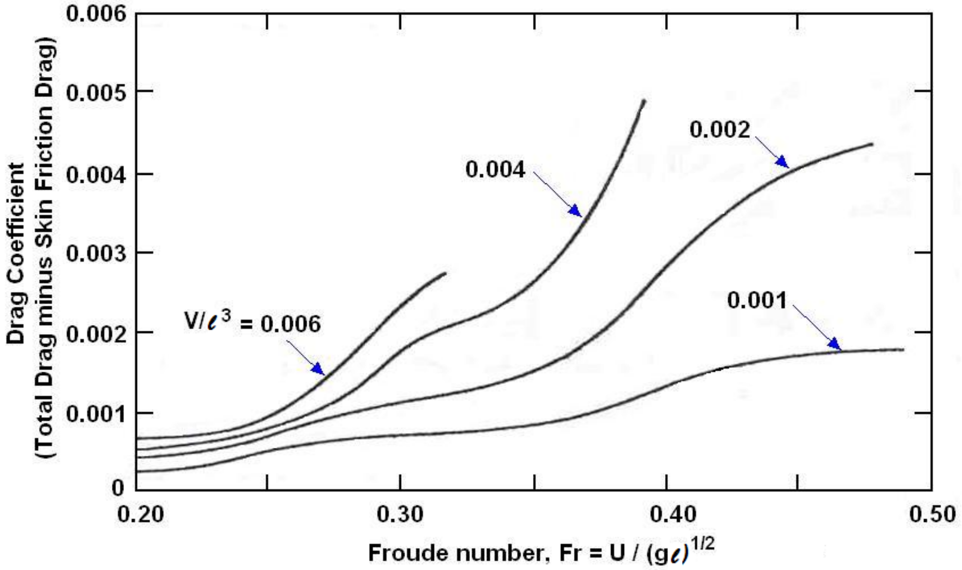

Figure 3: The coefficient of drag based on the wetted-hull area as a function of Froude number, $Fr = U/(g\ell)^{\frac{1}{2}}$, for four surface vessels with different ratios of submerged volume, V, to length, ℓ, cubed. The drag coefficient plotted is the total drag minus the skin friction drag.

on a deep ocean is given by $(g\lambda/2\pi)^{1/2}$; it follows that waves with a wavelength, λ, equal to the length of the ship, ℓ, will travel at the *same speed* as the ship if the Froude number, $Fr = 0.4$. Since the transverse waves are primarily created by the flow at the bow and the stern, it is these waves with $\lambda = \ell$ at $Fr = 0.4$ that will dominate the wave production process. It can be seen in Figure 3 that the wave drag begins to rise substantially at about $Fr = 0.4$. Of course other harmonics and sub-harmonics also play a role. Waves with $\lambda = \ell/2$ will occur at $Fr = 0.28$ and some effect at that Froude number can also be detected in Figure 3.

Before leaving the subject of ship wave drag, it is worth reiterating that the wave drag is the result of

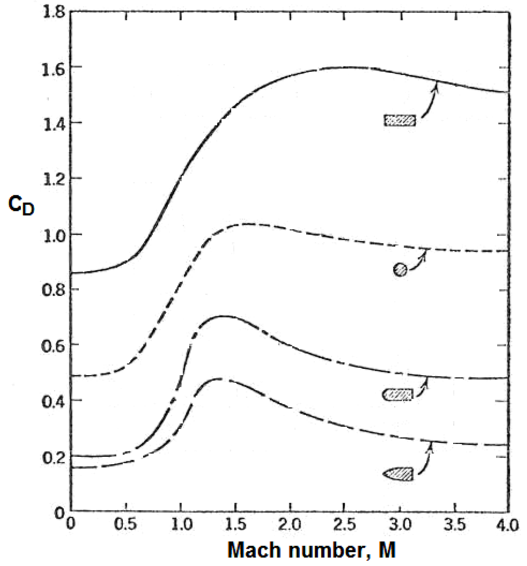

Figure 4: The coefficient of drag based on frontal projected area as a function of Mach number, $M = U/c$, for four different projectile shapes as shown.

the formation of waves which then radiate away from the ship. The energy lost by radiation is originally created by the additional force (equal and opposite to the drag) imposed by the ship on the ocean in order to create the waves. This is essentially the same mechanism that leads to the increase in the drag in subsonic compressible flow as the Mach number is increased; in that context the waves are acoustic. The effect on the drag in that case is shown in Figure 4.