Time Domain Methods

The application of time domain methods to one-dimensional fluid flow normally consists of the following three components. First, one establishes conditions for the conservation of mass and momentum in the fluid. These may be differential equations (as in the example in the next section) or they may be jump conditions (as in the analysis of a shock). Second, one must establish appropriate thermodynamic constraints governing the changes of state of the fluid. In almost all practical cases of single-phase flow, it is appropriate to assume that these changes are adiabatic. However, in multiphase flows the constraints can be much more complicated. Third, one must determine the response of the containing structure to the pressure changes in the fluid.

The analysis is made a great deal simpler in those circumstances in which it is accurate to assume that both the fluid and the structure behave barotropically. By definition, this implies that the change of state of the fluid is such that some thermodynamic quantity (such as the entropy) remains constant, and therefore the fluid density, $\rho(p)$, is a simple algebraic function of just one thermodynamic variable, for example the pressure. In the case of the structure, the assumption is that it deforms quasistatically, so that, for example, the cross-sectional area of a pipe, $A(p)$, is a simple, algebraic function of the fluid pressure, p. Note that this neglects any inertial or damping effects in the structure.

The importance of the assumption of a barotropic fluid and structure lies in the fact that it allows the calculation of a single, unambiguous speed of sound for waves traveling through the piping system. The sonic speed in the fluid alone is given by c_∞ where

$$c_\infty = (d\rho/dp)^{-\frac{1}{2}} \qquad \text{(Bnfa1)}$$

In a liquid, this is usually calculated from the bulk modulus, $\kappa = \rho/(d\rho/dp)$, since

$$c_\infty = (\kappa/\rho)^{-\frac{1}{2}} \qquad \text{(Bnfa2)}$$

However the sonic speed, c, for one-dimensional waves in a fluid-filled duct is influenced by the compressibility of both the liquid and the structure:

$$c = \pm\left[\frac{1}{A}\frac{d(\rho A)}{dp}\right]^{-\frac{1}{2}} \qquad \text{(Bnfa3)}$$

or, alternatively,

$$\frac{1}{\rho c^2} = \frac{1}{\rho c_\infty^2} + \frac{1}{A}\left(\frac{dA}{dp}\right) \qquad \text{(Bnfa4)}$$

The left-hand side is the acoustic impedance of the system, and the equation reveals that this is the sum of the acoustic impedance of the fluid alone, $1/\rho c_\infty^2$, plus an "acoustic impedance" of the structure given by $(dA/dp)/A$. For example, for a thin-walled pipe made of an elastic material of Young's modulus, E, the acoustic impedance of the structure is $2a/E\delta$, where a and δ are the radius and the wall thickness of the pipe ($\delta \ll a$). The resulting form of equation (Bnfa4),

$$c = \left[\frac{1}{c_\infty^2} + \frac{2\rho a}{E\delta}\right]^{-\frac{1}{2}} \qquad \text{(Bnfa5)}$$

is known as the Joukowsky water hammer equation. It leads, for example, to values of c of about $1000 m/s$ for water in standard steel pipes compared with $c_\infty \approx 1400 m/s$. Other common expressions for c are those used for thick-walled tubes, for concrete tunnels, or for reinforced concrete pipes (Streeter and Wylie 1967).