Combinations of Transfer Matrices

When components are connected in series, the transfer matrix for the combination is clearly obtained by multiplying the transfer matrices of the individual components in the reverse order in which the flow passes through them. Thus, for example, the combination of a pump with a transfer matrix, $[TA]$, followed by a discharge line with a transfer matrix, $[TB]$, would have a system transfer matrix, $[TS]$, given by

$$[TS] = [TB]\,[TA] \tag{Bnge1}$$

The parallel combination of two components is more complicated and does not produce such a simple result. Issues arise concerning the relations between the pressures of the inlet streams and the relations between the pressures of the discharge streams. Often it is appropriate to assume that the branching which creates the two inlet streams results in identical fluctuating total pressures at inlet to the two components, $\tilde{p}_1^T$. If, in addition, mixing losses at the downstream junction are neglected, so that the fluctuating total pressure, $\tilde{p}_2^T$, can be equated with the fluctuating total pressure at discharge from the two components, then the transfer function, $[TS]$, for the combination of two components (order two transfer functions denoted by $[TA]$ and $[TB]$) becomes

$$\begin{aligned}
TS_{11} &= (TA_{11}TB_{12} + TB_{11}TA_{12})/(TA_{12} + TB_{12}) \\
TS_{12} &= TA_{12}TB_{12}/(TA_{12} + TB_{12}) \\
TS_{21} &= TA_{21} + TB_{21} - (TA_{11} - TB_{11})(TA_{22} - TB_{22})/(TA_{12} + TB_{12}) \\
TS_{22} &= (TA_{22}TB_{12} + TB_{22}TA_{12})/(TA_{12} + TB_{12})
\end{aligned} \tag{Bnge2}$$

On the other hand, the circumstances at the junction of the two discharge streams may be such that the fluctuating static pressures (rather than the fluctuating total pressures) are equal. Then, if the inlet static pressures are also equal, the combined transfer matrix, $[TS^*]$, is related to those of the two components ($[TA^*]$ and $[TB^*]$) by the same relations as given in equations (Bnge2). Other combinations of choices are possible, but will not be detailed here.

Using the above combination rules, as well as the relations (Bngc7) between the $[T]$ and $[T*]$ matrices, the transfer functions for very complicated hydraulic networks can be systematically synthesized.