Rough Pipe Flows

In section (Bkl), turbulent flow in smooth pipes was investigated using both the universal turbulent velocity profile and a cruder velocity profile due to Blasius. The result was prediction of the dependence of the friction factor on the Reynolds number and shapes for the velocity profile(s) at various Reynolds numbers. To follow a parallel development for rough walled tubes we need only substitute the universal velocity profile for rough walls (equation (Bkj12)), namely

$$u^* \;=\; \frac{\overline{u}}{u_\tau} \;=\; C_3 \; log_{10}(y/\epsilon) + C_4 \tag{Bkm1}$$

(where estimated values of C_3 and C_4 are approximately 5.75 and 8.5 respectively) and follow the same procedure. This yields the following relation for the friction factor, f, in terms of the relative roughness, ϵ/R:

$$\frac{1}{f^{\frac{1}{2}}} \;=\; -2 \; log_{10}\left\{\frac{\epsilon}{2R}\right\} + 1.0 \tag{Bkm2}$$

where, as before, the last constant depends somewhat on the data used to establish the integration constant. Note that there is no dependence on the Reynolds number since neither the Reynolds number nor the viscosity appear in the turbulent velocity profile. Consequently, when the results of this analysis are plotted in the Moody diagram, they appear as a set of horizontal lines, one for each relative roughness, ϵ/R, as shown in Figure 1 or 2.

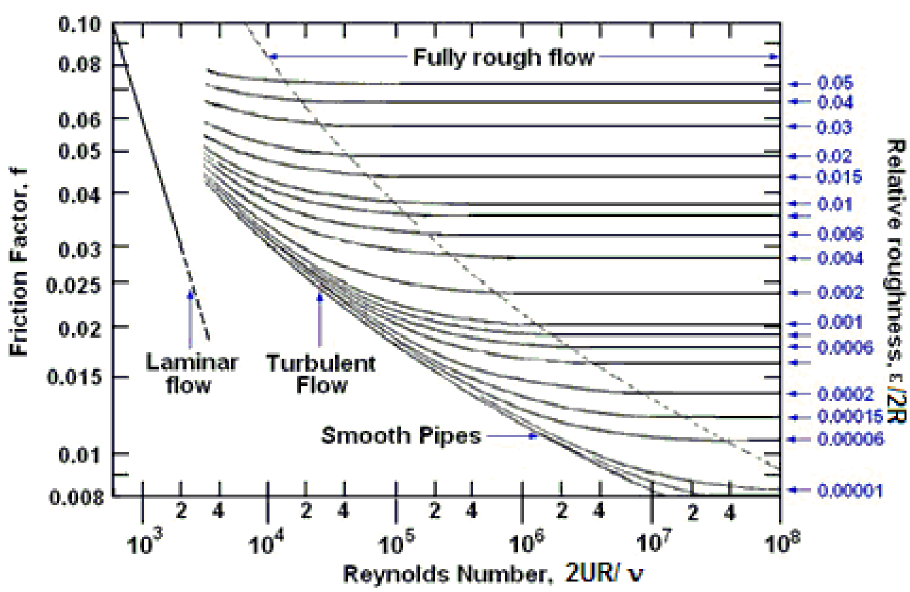

Figure 1: The Moody diagram, the friction factor, f, for flow in a circular pipe plotted against the Reynolds number, $2VR/\nu$.

However, the universal profile of equation (Bkm1) and the results which follow from it, assume that the pipe wall is “fully rough”. That is to say they assume that the roughness, ϵ, is larger than the laminar sublayer that would have been present without the roughness. To evaluate this we recall that this potential

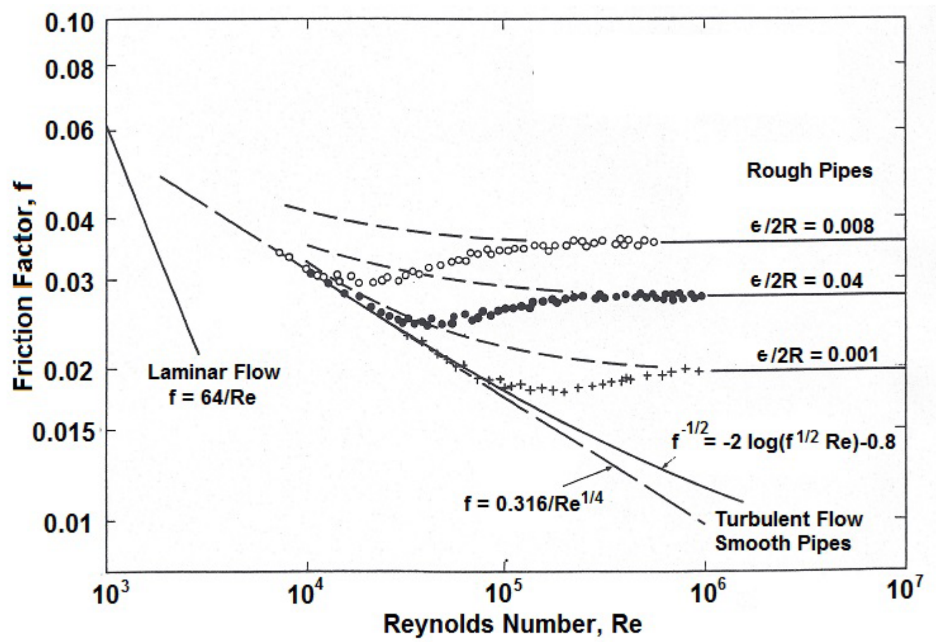

Figure 2: Moody diagram of friction factor, f, for a circular pipe plotted against Reynolds number, $2RV/\nu$.

laminar sublayer thickness, δ_{LSL}, would be given by equation (Bkj7), namely

$$\delta_{LSL} \;=\; \frac{5\nu}{(\tau_W/\rho)^{\frac{1}{2}}} \;=\; \frac{20R}{Re(f/2)^{\frac{1}{2}}} \tag{Bkm3}$$

using the definition of the friction factor, equation (Bkl6), and the Reynolds number, $Re = 2RV/\nu$. To evaluate this further, it is convenient to use the relation (Bkl13) for the friction factor in smooth pipes,

$$f \;=\; \frac{0.309}{Re^{1/4}} \tag{Bkm4}$$

to approximately evaluate δ_{LSL} in the potential smooth wall case using

$$\frac{\delta_{LSL}}{2R} \;\approx\; 25Re^{-\frac{7}{8}} \tag{Bkm3}$$

This then defines the boundary between a transitional regime in which there may be some laminar boundary layer and the "fully rough" regime in which there is no laminar boundary layer and the friction factor is independent of the Reynolds number, Re. This boundary is indicated by the dashed line in Figure 1. The curves in Figure 1 for the transitional regime that asymptote to the curve for smooth pipes at the lower Reynolds numbers and to the lines for the fully rough pipes at the higher Reynolds numbers were heuristically constructed. However, the actual experimentally measured transition is a little more complex as indicated by the data plotted in Figure 2.