# Hydraulic System Analysis

Using the tools of continuity and Bernoulli's equation (modified by estimated viscous loss coefficents where appropriate) the fluids engineer can proceed with the analysis of steady, incompressible flow through many hydraulic systems. The simplest and most widely used approach is to assume that the flow at every cross-section of the incompressible internal flow can be characterized by a single pressure, $p$ (or total pressure, $p^T$), and a single mass flow rate, $m$, (or volume-averaged velocity, $u$). This approach will be termed the one-dimensional analysis of hydraulic systems. Details such as the velocity profile of the flow at any location, are set aside and the loss coefficients are assumed to be functions only of the mass flow rate. Those details (and, in some cases, their effects upon the accuracy of the one-dimensional analysis) are addressed later during investigations of the two- and three-dimensional fluid mechanics.

Assuming some recourse to identifying the loss coefficients in all of the components of the system (see, for example, Moody (1944), Idelchik (1994), Crane (1957)), such an analysis, in its simplest form, would proceed as follows:

1. The system is subdivided into its component parts, each identified by its index, $k$, as shown in figure 1 where each component is represented by a box. The connecting lines do not depict lengths of pipe

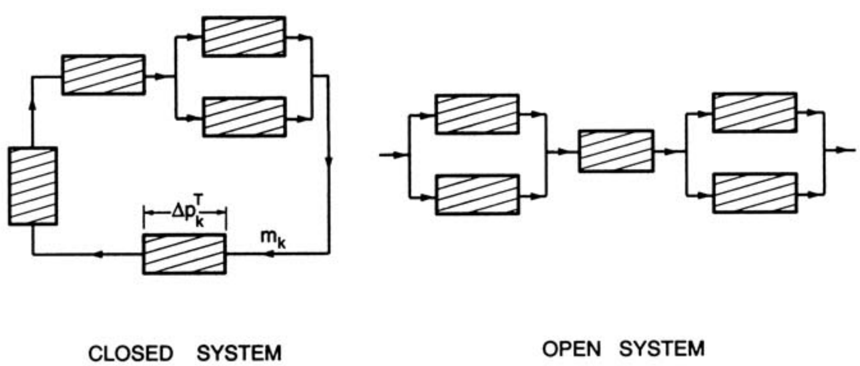


Figure 1: Hydraulic systems broken into components.

which are themselves components. Rather the lines simply show how the components are connected. More specifically they represent specific locations at which the system has been divided up; these points will be called the nodes of the system and are denoted by the index, $i$. Typical and common components are pipeline sections, valves, pumps, turbines, boilers, and condensers. They can be connected in series and/or in parallel. Systems can be either open loop or closed loop as shown in figure 1. The mass flow rate through a component will be denoted by $m_k$ and the change in the total pressure (or, equivalently, the total head) across the component will be denoted by $\Delta p_k^T$ (or $\Delta H_k$) defined as the total pressure (or head) at inlet minus that at discharge.

2. Next, the performance characteristic of the components considered in isolation needs to be identified. The performance characteristic is the relation between the total pressure drop (or total head drop) and the mass flow rate, namely the function $\Delta p_k^T(m_k)$ (or $\Delta H_k(m_k)$) as depicted graphically in figure 2. Some of these performance characteristics are readily anticipated. For example, a typical steady, incompressible flow through a horizontal pipe or passive fitting has a characteristic that is approximately quadratic (at least at high Reynolds number) with $\Delta p_k^T \propto m_k^2$. By definition of the

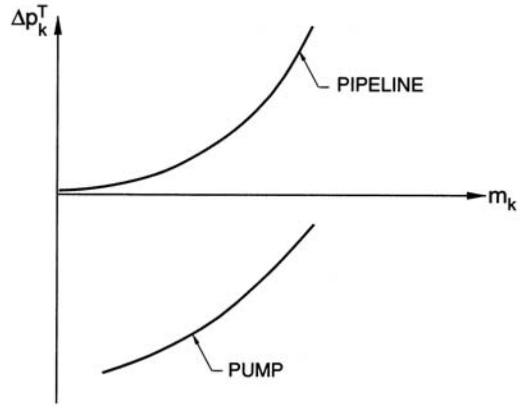


Figure 2: Typical component characteristics, $\Delta p_k^T(m_k)$.

loss coefficient, $K_k$,

$$\Delta p_k^T = \left\{ \frac{K_k}{2\rho^2 A_k^2} \right\} m_k^2 \quad \text{or} \quad \Delta H_k = \left\{ \frac{K_k}{2g\rho^3 A_k^2} \right\} m_k^2 \tag{Bfa1}$$

Other components such as pumps, compressors or fans may have quite non-monotonic characteristics. The slope of the characteristic, $R_k^*$, where

$$R_k^* = \frac{d\Delta H_k}{dm_k} = \frac{1}{\rho g}\frac{d\Delta p_k^T}{dm_k} \tag{Bfa2}$$

is known as the component resistance. However, unlike many electrical components, the resistance of most hydraulic components is almost never constant but varies with the flow, $m_k$.

3. Components can then be combined to obtain the characteristic of groups of neighboring components or of the complete system. A parallel combination of two components simply requires one to add the flow rates at the same $\Delta p^T$ (or $\Delta H$), while a series combination simply requires that one add the $\Delta p^T$ values of the two components at the same flow rate. In this way one can synthesize the total pressure drop, $\Delta p_s^T(m_s)$, for the whole system as a function of the system flow rate, $m_s$ (into and out of the system). Such a system characteristic is depicted in figure 3.

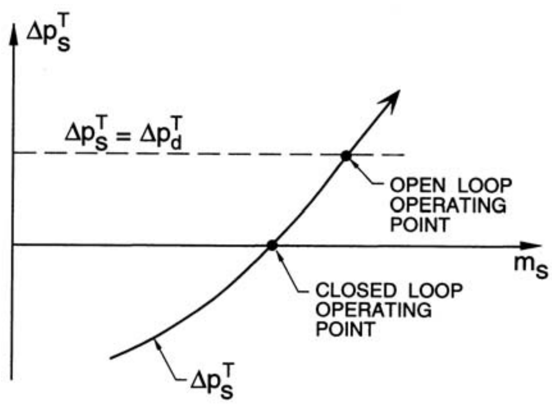


Figure 3: Typical system characteristic, $\Delta p_s^T(m_s)$, and operating point.

4. The final step is to apply whatever known boundary condition is appropriate for the whole system. Usually (but not always) this consists of a known system total pressure difference, $\Delta p_s^T$, and it is

this which determines the operating point of the system. Commonly, when the boundary condition is a known total pressure difference, the result is the determination of the system flow rate; more generally, the overall boundary condition determines the system operating point. For a closed system, the equilibrium operating point is given by the intersection of the characteristic with the horizontal axis since the closure boundary condition is $\Delta p_s^T = 0$. An open system driven by a known total pressure difference of $\Delta p_d^T$ (inlet total pressure minus discharge) would have an operating point where the characteristic intersects the horizontal line at $\Delta p_s^T = \Delta p_d^T$ as indicated in Figure 3.

It will be useful to provide an example of such an analysis and we choose the system shown in Figure

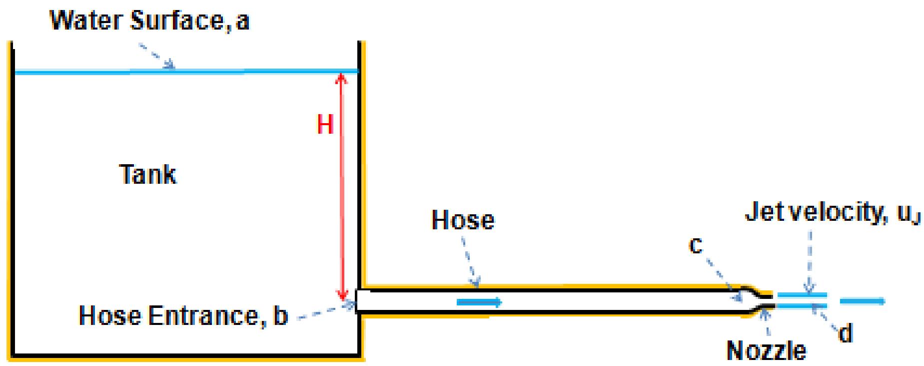


Figure 4: Example consisting of a tank, hose and nozzle.

4 consisting of a tank, hose and nozzle. The tank has a very large surface area denoted by the location index, $i = a$. That surface is at atmospheric pressure, $p_a$, and is at an elevation $y = H$ above the entrance to the hose. That entrance is denoted by the location index, $i = b$, and we choose that to be the reference elevation, $y = 0$. The hose is assumed to be horizontal, to have an internal cross-sectional area, $A_H$ and a loss coefficient, $K_H$. If we denote the mass flow rate by $m$ then the volume-averaged velocity in the hose is $u_H = m/\rho g A_H$, where $\rho$ is the incompressible fluid density. The last component is the nozzle on the end of the hose; the node at the junction of the hose and the nozzle is given the location index $i = c$ and the jet emerging from the nozzle is $i = d$. The jet cross-sectional area is $A_J$ so the jet velocity, $u_J = A_H u_H / A_J$. Finally we denote the loss coefficient for the nozzle (based on $u_H$) by $K_N$. Then, beginning at the tank surface and working our way through the system we can write the following relations for the total pressures, $p^T$, at each of the nodes as

$$\begin{aligned}
p_a^T &= p_a + \rho g H \quad \text{(neglecting the dynamic head since the velocity is very small)} \\
p_b^T &= p_a^T \quad \text{(assuming no loss in tank since the velocity is very small)} \\
p_c^T &= p_b^T - \frac{1}{2} u_H^2 K_H \quad \text{(accounting for the loss in the hose)} \\
p_d^T &= p_c^T - \frac{1}{2} u_H^2 K_N \quad \text{(accounting for the loss in the nozzle)} \\
&= p_a + \frac{1}{2} u_J^2 \;=\; p_a + \frac{u_H^2 A_H^2}{A_J^2}
\end{aligned}$$

where the last equation is the application of the system boundary condition, namely the known total pressure difference across the system. Eliminating the intermediate quantities yields the result that

$$m \;=\; \rho u_H A_H \;=\; \rho \left[ \frac{2gH}{\left\{ \frac{1}{A_J^2} + \frac{(K_H + K_N)}{A_H^2} \right\}} \right]^{\frac{1}{2}} \qquad \text{(Bfa3)}$$

and

$$u_J = \left[ \frac{2gH}{\left\{1 + \frac{(K_H+K_N)A_J^2}{A_H^2}\right\}} \right]^{\frac{1}{2}} \quad \text{(Bfa4)}$$

Note that, as would be expected, the mass flow rate and the associated jet velocity decrease with increase in the hose and nozzle loss coefficients. Note also that the jet velocity decreases as the jet to hose area ratio, $A_J/A_H$ is increased, an effect known to any gardener.

The procedure outlined above depends on the loss coefficents, $K_k$, being known. When these are themselves functions of the flow rate $m_k$, the solution becomes more complicated. This is often the case when the loss coefficient is a function of the Reynolds number, $Re = UD/\nu = mD/\rho\nu A$, and thus a function of the mass flow rate as in the case of lengths of pipe. Then, one must start with an estimate the Reynolds number (or the loss coefficient), proceed through steps 1 to 4, evaluate the flow rate, correct the Reynolds number and continue to cycle through this procedure until the results converge.