Planar Doublet

A doublet consists of a source and a sink of equal magnitude, Q, and distance a apart which are brought

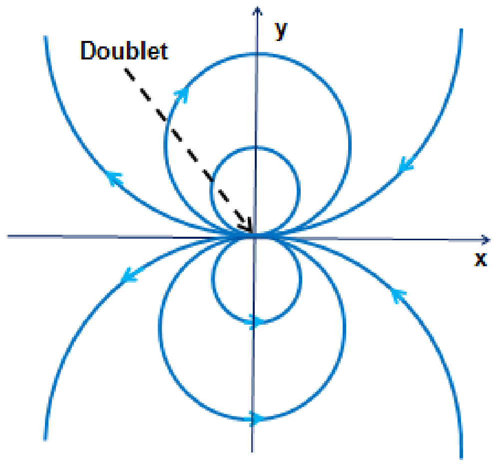

Figure 1: Streamlines due to a planar doublet oriented in the x direction.

together so that Qa is a constant while $a \to 0$ and $Q \to \infty$. The flow pattern produced depends on the orientation of the source and sink. If these are on the x axis, the streamlines generated are as depicted in Figure 1; if they had been placed on the y axis the flow pattern would simply be rotated by $\pi/2$. It follows that a doublet has two identifying features, namely the strength, Qa, and the orientation. Consider a planar potential line doublet of strength, Qa, oriented on the x axis. From equations (Bgdf1) and (Bgdf2), namely,

$$\phi = \frac{Q}{4\pi} \ln \left\{ \frac{(x+a)^2+y^2}{(x-a)^2+y^2} \right\} \tag{Bgdf1}$$

$$\psi = \frac{Q}{2\pi} \left[\arctan\left\{ \frac{y}{(x+a)} \right\} - \arctan\left\{ \frac{y}{(x-a)} \right\} \right] \tag{Bgdf2}$$

it follows that if a is very small then

$$\phi = \frac{Q}{4\pi} \ln \left\{ 1 + \frac{4ax}{(x-a)^2+y^2} \right\} \approx \frac{Q}{4\pi} \frac{4ax}{(x^2+y^2)} = \frac{Qa}{\pi} \frac{x}{(x^2+y^2)} = \frac{Qa}{\pi} \frac{\cos\theta}{r} \tag{Bgdf3}$$

where the polar coordinates, r and θ, are $x = r\cos\theta$ and $y = r\sin\theta$ as before. Consequently, provided Qa remains constant, we can allow a to approach zero and thus generate a doublet with this velocity potential.

Several notes are appropriate at this point. First note that the strength, Qa, can be positive or negative depending on whether the source is in front of or behind the sink. Note also that when the doublet is oriented in the y direction rather than the x direction the $\cos\theta$ is replaced by a $\sin\theta$.

It is also of interest to observe that the velocity potential in equation (Bgdf3) is simply the x derivative of the potential due to a simple source. This is not surprising when one recognizes that if ϕ is a solution of Laplace's equation, $\nabla^2\phi = 0$ then $\partial\phi/\partial x$ must also be a solution of Laplace's equation (as must $\partial\phi/\partial y$ which is the doublet oriented in the y direction). Indeed, any higher derivatives of ϕ must be a solution and so we see that there is an infinite array of solutions resulting from repeated derivatives of the simple source. Moreover, we should also note that the potential due to the doublet dies off with distance, r, like $1/r$ which is faster than the rate at which the basic source potential dies off which is like $\ln r$. Second derivatives would die off even faster, like $1/r^2$, etc. The upshot is that if we seek to generate a potential flow by superimposing multiple singularities then those higher derivatives will influence the nearby flow but have little effect on the distant flow.

A related feature is that if we wish to create the flow about a finite body or object then the sum of the strengths of the sources used must equal the sum of the strengths of the sinks for otherwise there will be a net volume flow into or out of the near-field region. One way to ensure that the generated body is finite is to use only doublets and higher order singularities and this is a commonly used strategy.