The Free Vortex

One of the simplest of the potential flows due to a singularity is that of a **free vortex** given by

$$\phi = C\theta \quad ; \quad u_r = 0 \quad ; \quad u_\theta = \frac{C}{r} \quad ; \quad \psi = -C \ln r \tag{Bgdc1}$$

which clearly corresponds to the solution with $C_{20}C_{30} = C$ and with all the other C coefficients equal to zero. This is a flow with circular streamlines and a circumferential velocity, u_θ, which decreases like $1/r$ with increasing distance, r, from the origin. Examination of a small fluid element in this flow shows that though it is being highly distorted by the flow it does not rotate as it shouldn't since the flow is irrotational.

However, if we evaluate the circulation around any circle about the origin $r = 0$ we find

$$\Gamma = \int_0^{2\pi} u_\theta r d\theta = 2\pi C \tag{Bgdc2}$$

which appears contradictory since the flow is irrotational and $\omega = 0$. To investigate this seeming contradiction we could evaluate the circulation around any other contour that did not include the origin and we would find that, as expected, that circulation is indeed zero. Thus we conclude that there is something special or "singular" about the origin, that it is a point of infinite vorticity that has a finite contribution to the circulation around any contour that includes the origin. Indeed it is conventional to characterize the strength of a free vortex by the magnitude of the circulation, Γ, rather than the constant, C. Thus a free vortex is normally described by

$$\phi = \frac{\Gamma}{2\pi}\theta \quad ; \quad u_r = 0 \quad ; \quad u_\theta = \frac{\Gamma}{2\pi r} \quad ; \quad \psi = -\frac{\Gamma}{2\pi} \ln r \tag{Bgdc3}$$

The free vortex is not to be confused with a **forced vortex** which is defined as a flow with circular streamlines in which the velocity, u_θ, increases with distance from the origin and is proportional to r. This forced vortex is not a potential flow. The motion is often referred to as solid body rotation.

A real vortex cannot, of course, include the infinite velocity at the origin which a free vortex exhibits.

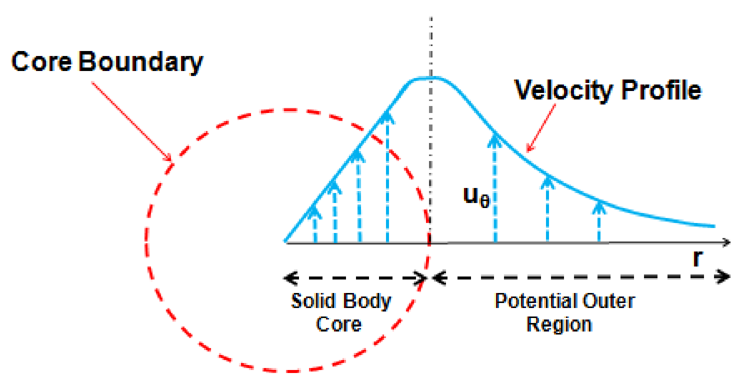

Figure 1: Real vortex with viscous core and potential outer flow.

What happens in real vortices is that the shear near the origin becomes so great that viscous forces come into play and prevent the shear from reaching such high levels. What this does is to create a core in the center of the vortex that is dominated by viscous effects and that produces a force vortex in the center. Thus a real vortex often takes the form sketched in Figure 1 in which there is an inner core of solid body rotation in which the velocity is proportional to r. This is surrounded by an outer potential flow or free vortex with a velocity proportional to $1/r$ so that the velocity goes to zero as $r \rightarrow \infty$.