Gravity Waves at Vertical Boundaries

In this section we consider the interaction between gravity waves and a solid object that intersects the liquid surface. For the sake of simplicity, we will focus on a simple vertical wall as depicted in Figure 1. The boundary condition at this wall will therefore be that the horizontal velocity, u, must be zero at all times. But this is not a possibility within the traveling wave solutions. It is however possible to locate a vertical wall at one of the x locations in the standing wave solutions of equations (Bgce5) to (Bgce7) or (Bgce8) to (Bgce10) at which u is zero at all times, namely at $kx = \pi/2$, $3\pi/2$, etc. or $kx = -\pi/2$, $-3\pi/2$, etc. This allows the construction of a series of solutions of waves interacting with vertical walls or waves sloshing back and forth in a box. A few examples will illustrate the possibilities.

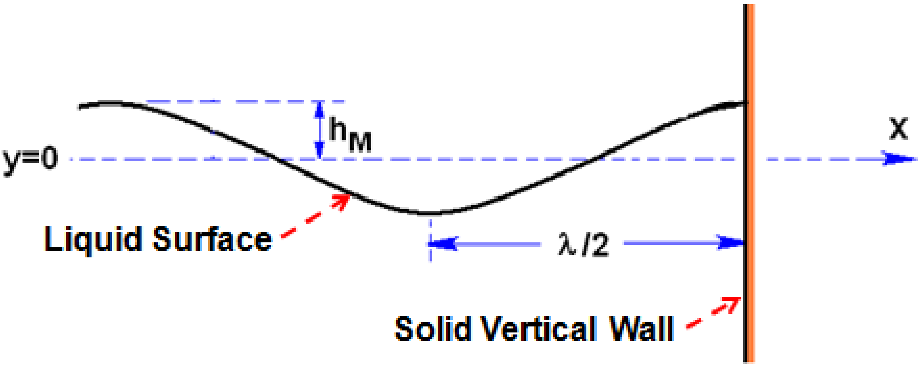

Figure 1: Gravity waves at a solid, vertical wall.

The first and simplest example involves a single vertical wall as shown in Figure 1. If we set out to seek the solution for a train of travelling waves (propagating in the positive x direction) we would not be able to find a solution which satified the condition of $u = 0$ at the solid wall. We would need to add a set of waves of equal amplitude traveling in the negative x direction and thus transforming the surface to a set of standing waves as illustrated in Figure 2. One could imagine that this additional set of waves represents the reflection of the initial set of waves from the solid wall. Note that the amplitude of the standing waves is twice the amplitude of the component traveling wave trains. This is analgous to the observation of sea waves impacting a harbor wall and the way the waves climb the wall to almost twice their incoming amplitude.

Another example is waves sloshing back and forth between two vertical walls as depicted in Figure 3. If the liquid between is infinitely deep then the solution is simply given by equations (Bgce5) to (Bgce7) and the first mode of sloshing (Figure 3, left) is then given by placing the vertical walls one wavelength apart while the second mode is produced by placing the vertical walls two wavelengths apart (Figure 3, right) and so on to as many wavelengths as desired. It follows that if the distance between the vertical walls is L then the various modal wavelengths, λ_n where $n = 1, 2, 3, ...$ are given by $\lambda_n = L/n$ and, from equation (Bgce10), the frequencies, ω_n, of each of these modes are given by

$$\omega_n = (2\pi g n/L)^{\frac{1}{2}} \qquad \text{(Bgcf1)}$$

Note that the lowest mode, $n = 1$, has the lowest frequency, $\omega_1 = (2\pi g/L)^{\frac{1}{2}}$ and that the frequencies

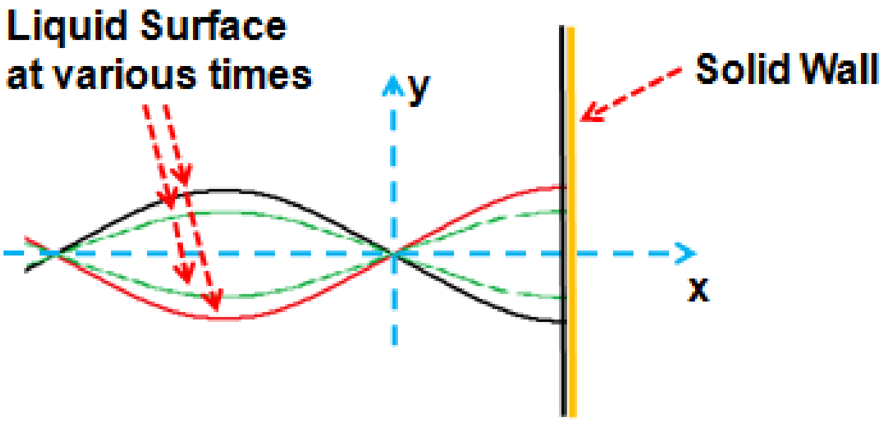

Figure 2: The standing waves next to a solid vertical boundary.

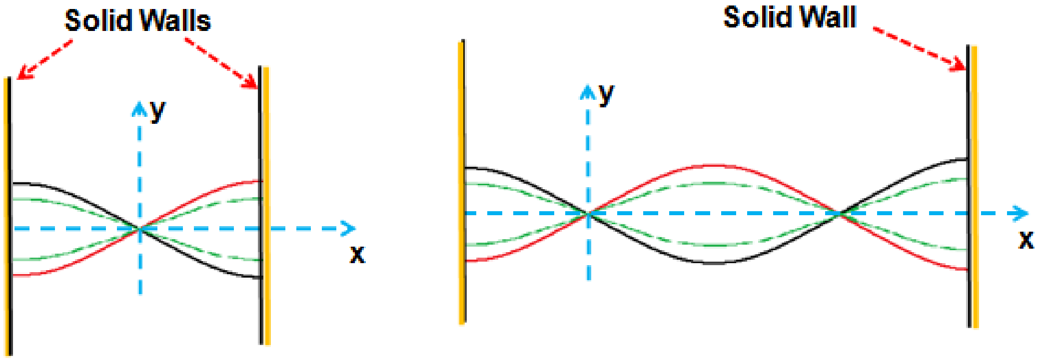

Figure 3: Waves sloshing between two vertical walls: first mode (left) and second mode (right).

of higher modes increase with the square root of the number of wavelengths manifest. Note also that the frequencies decrease with the width, L.

We can, of course, examine the modes of sloshing in a rectangular box (Figure 4) in a similar way by inserting vertical walls into the standing wave solution for waves on a liquid of finite depth, equations (Bgce8) to (Bgce10). As in the case of infinite depth we denote the distance between the vertical walls by L and the various modal wavelengths, by λ_n where $n = 1, 2, 3, ...$ and $\lambda_n = L/n$. Then, using equation

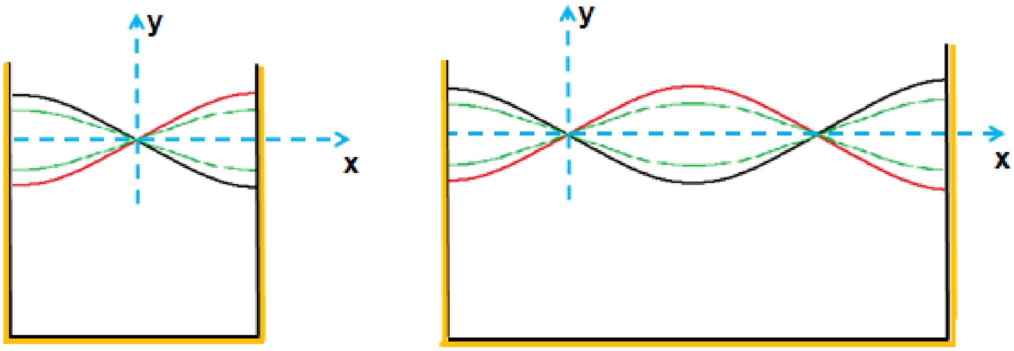

Figure 4: Waves sloshing in a rectangular box: first mode (left) and second mode (right).

(Bgcf1) for the modal frequencies, ω_n, we find

$$\omega_n = \left\{ \frac{2\pi g n}{L} \tanh \frac{2\pi H}{L} \right\}^{\frac{1}{2}} \tag{Bgcf2}$$

Note that the frequencies decrease as the aspect ratio of the box, H/L, decreases. Sloshing can be a major problem in large liquid tanks subjected to imposed motion, particularly if the frequencies of the imposed motion correspond to these natural frequencies of sloshing. For example, this can be a serious issue for swimming pools, fuel tanks and water tanks in earthquake country. It can also be a serious issue in ships with tanks designed to transport liquids.