Introduction to Open Channel Flow

Open channel flows refer to liquid flows that are confined to rivers or open channels by the action of gravity. They involve an upper free surface exposed to a gaseous environment (or vacuum) and may therefore feature the propagation of waves. Consequently, before more detailed analyses of open channel flows we need to begin by establishing some necessary background on wave propagation at a free surface. The first installments of this essential background are to be found in sections (Bgcd) to (Bgcf) which are also listed in the index for this major topic. In section (Bgcd) the propagation speed, c, of small amplitude waves of wavelength, λ, in an ocean of depth, H, was derived within the context of incompressible, irrotational flow, namely

$$c = (g\lambda \tanh(2\pi H/\lambda)/2\pi)^{\frac{1}{2}} \quad \text{(Bpa1)}$$

where g is the acceleration due to gravity. When the wavelength, λ, is much smaller than the depth, H, the waves are referred to as *deep water waves* and the propagation speed becomes

$$c \rightarrow (g\lambda/2\pi)^{\frac{1}{2}} \quad \text{(Bpa2)}$$

In contrast when $\lambda \gg H$, the waves are known as *shallow water waves* and the propagation speed is:

$$c \rightarrow (gH)^{\frac{1}{2}} \quad \text{(Bpa3)}$$

In open channel flow we will be primarily concerned with shallow water waves.

An alternative derivation of the shallow water wave propagation speed can be obtained from a simple one-dimensional analysis of an open-channel flow. As depicted in Figure 2, we select a control volume that spans the full depth, $H(x)$, of the open channel flow but with a dimension dx in the direction of the flow. We make the assumption that the variations in the flow characteristics (velocity, density) with depth can be neglected so the flow is completely characterized by the functions, $H(x)$, and the depth-averaged velocity, $u(x)$, in the x-direction. For simplicity, unit breadth normal to Figure 2 will be assumed. We will examine the velocity of a small disturbance propagating with velocity, c, into a fluid at rest as depicted on the left in Figure 2. However, it is convenient to make a Galilean transformation to a frame in which the propagating wave is fixed as shown on the right in Figure 2. Then conservation of mass requires that

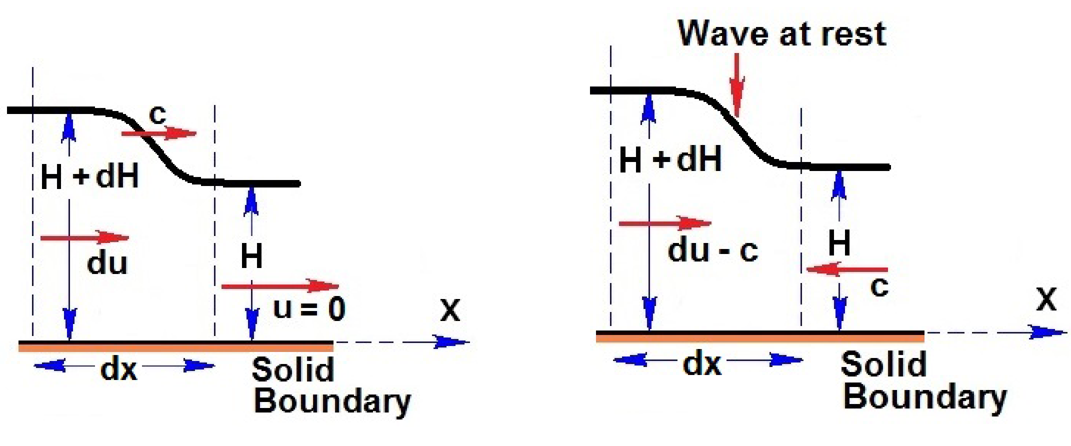

Figure 1: Small disturbance propagation: Left: In frame of right-hand fluid. Right: In frame fixed in wave.

$$Hc = (c - du)(H + dH) \quad \text{or} \quad c\,dH = H\,du \quad \text{(Bpa4)}$$

To apply the linear momentum equation to the control volume we must first evaluate the net force acting on the control volume in the x direction. The force due to the hydrostatic pressure acting on the right side of the control volume is $\rho g H^2/2$ while that on the left side is $\rho g(H+dH)^2/2$ so that the net hydrostatic force is $\rho g H dH$ in the positive x direction (neglecting the terms quadratic in dH). We neglect any viscous forces that may act at the solid boundary (see section (Bpe)). The momentum flux in through the lefthand boundary is $-\rho(c-du)^2(H+dH)$ and out through the right-hand boundary is $\rho c^2 H$. Consequently the linear momentum theorem yields

$$\rho c^2 H - \rho(c-du)^2(H+dH) \;=\; \rho c H du \;=\; \rho g\ H\ dH \qquad \text{or} \qquad c\ du \;=\; g\ dH \tag{Bpa5}$$

where we have neglected all terms that are quadratic in the small quantities, du, dH. Eliminating du from equations (Bpa4) and (Bpa5) leaves

$$\rho c^2 H - \rho(c-du)^2(H+dH) \;=\; \rho c H du \;=\; \rho g\ H\ dH \qquad \text{or} \qquad c^2 \;=\; gH \tag{Bpa6}$$

as in equation (Bpa3). In the sections that follow we will present similar analyses that incorporate finite amplitude disturbances and that include viscous forces acting on the bottom of the control volume. We note that typical velocities, c, are commensurate with our everyday experience with open channel flows. For example, the wave velocity in a $1m$ deep open channel flow is just over $3m/s$.

It follows that open channel flows with the wave propagation velocity of $(gH)^{1/2}$ are quite analogous to compressible flows with wave velocity equal to the speed of sound. We define a parameter analogous to the Mach number that is called the *Froude number*, $Fr = u/(gH)^{1/2}$, so that open channel flows with $Fr < 1$ are termed *subcritical flows* while those with $Fr > 1$ are termed *supercritical flows*. Supercritical flows are characterized by the fact that disturbances cannot propagate upstream. In the next section we investigate compression waves and expansion waves in open channel flow much as we studied these processes in the context of compressible flows in section (Boe). We also remark that the equivalent of shock waves are the tidal bores illustrated in Figures 2 and 3, hydraulic jumps and other open channel shock waves.

Figure 2: Tidal bore in Morecambe Bay, UK. Photograph by Arnold Price.

Figure 3: Tidal bore in Moncton, Bay of Fundy, Canada. Photograph by Charles LeGresley.