# Frictionless Flow

Having established some of the basic features of open channel flows in flat-bottomed conduits, we now broaden the discussion to consider flows in channels with varying bottom topography. However, for simplicity, we begin with discussion of steady, planar flows in the absence of viscous effects. A typical bottom geometry is sketched in Figure 1 where $Z(x)$ is the elevation of the solid bottom as a function of the horizontal coordinate, $x$; $Z$ is most conveniently measured from a datum corresponding to the water level at a point where the depth is very large (effectively infinite). It is also convenient to consider a unit breadth of flow normal to the sketch in Figure 1. We begin by developing the equations for steady, planar, open

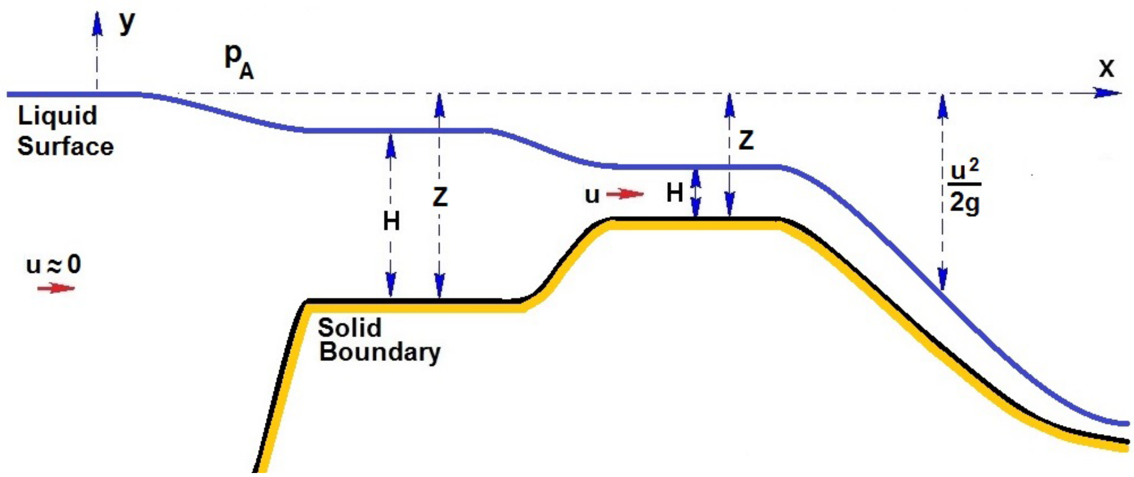


Figure 1: General planar open channel flow notation.

channel flow of an incompressible, inviscid fluid assuming that the any surface waves have a wavelength much greater than the depth so that the velocity, $u(x)$, can be assumed uniform over the depth. The liquid surface is assumed to be everywhere at a uniform atmospheric pressure, $p_A$, and surface tension effects are assumed negligible. We also note that viscous, frictional effects at the underlying boundary will be addressed in the section (Bpe) that follows. The basic governing equations that follow from these assumptions are

- *Continuity Equation:*

$$u(x)\ H(x)\ =\ Q \tag{Bpd1}$$

  where $Q$ is the volume flow rate per unit breadth normal to the above sketch and $Q$ is a simple constant.

- *Bernoulli Equation:*

$$p(x) + \frac{1}{2}\rho u^2 + \rho g y\ =\ p^T(x)\ =\ \text{constant} \tag{Bpd2}$$

  where $p(x, y)$ is the fluid pressure, $p^T(x)$ is the total pressure, $\rho$ is the fluid density, $g$ is the acceleration due to gravity and $y$ is a coordinate measured vertically upward. This will be valid along any streamline or streamtube provided no hydraulic jump is passed.

It is convenient and conventional to define a datum level, $y = 0$, at the elevation of the fluid surface where the velocity, $u(x)$, is negligible as depicted in Figure 1. Then, with the assumptions listed above, the elevation difference between the datum level, $y = 0$, and the elevation of the fluid surface is simply given by $y = -u^2/2g$. Moreover this quantity is also constant at any point within the fluid since the increase in the pressure, $p$, below the surface is matched by a corresponding decrease in the potential energy. Note also that the quantity $y + u^2/2g$ is called the *piezometric head* and, under the assumptions listed above, this quantity is constant throughout the fluid provided no hydraulic jump intervenes. In the section (Bpe) which follows, the effects of friction at the underlying solid surface are most readily incorporated by a downward sloping datum level that reflects the decrease in the Bernoulli constant caused by the frictional effects.

In the absence of friction, the constant piezometric head can be written as

$$\frac{Q^2}{2gH^2} + H - Z = \text{constant} \tag{Bpd3}$$

or

$$\left\{1 - \frac{Q^2}{gH^3}\right\} \frac{dH}{dx} = \frac{dZ}{dx} \tag{Bpd4}$$

Therefore, at a flat location where $dZ/dx = 0$,

- *either*:

$$\frac{dH}{dx} = 0 \tag{Bpd5}$$

- *or*:

$$\frac{Q^2}{gH^3} = 1 \qquad \text{in other words} \qquad Fr = 1 \tag{Bpd6}$$

The other conclusion to be drawn from equation (Bpd4) is that

$$\text{If} \qquad Fr = 1 \qquad \text{then} \qquad \frac{dZ}{dx} = 0 \tag{Bpd7}$$

It follows that a flat location where $dZ/dx = 0$ is just like a throat in compressible fluid flow. A Froude number of unity can only occur at a flat location though it is also possible at a flat location that $dH/dx = 0$. But we necessarily conclude that a flow can only transition from subcritical ($Fr < 1$) to supercritical ($Fr > 1$) at a flat location though it is also possible, depending on the conditions downstream, for a flow with $Fr = 1$ at the flat location to become either subcritical or supercritical downstream of the flat location. Very frequently, because of the downstream conditions, the flow becomes supercritical downstream of the flat location and the only way in which it can transition back to subcritical is by passing through a hydraulic jump. In these respects, open channel flow is very analogous to one-dimensional compressible flow.