Finite Amplitude Waves

Consider now the propagation of finite amplitude, shallow water waves. To aid in the evaluation, it is useful to consider a body of fluid of depth, H_0, which is initially at rest. A wall at one end is then moved into or out off the fluid so as to to produce, respectively, a compression or expansion wave that then propagates away from the wall as depicted in Figure 1. The radical difference between the compression and expansion cases becomes apparent when we consider the waves conceptually subdivided into a series of small disturbances as was done in the context of a compressible fluid in section (Boh). In both circumstances the first, small disturbance will travel at the propagation speed in the undisturbed fluid, namely $(gH_0)^{1/2}$. In the case of the expansion wave (as depicted in Figure 2), the next or following small perturbation will then propagate into shallower fluid, and will therefore travel at a slower speed than the first perturbation. It will be followed by an even slower perturbation and so the expansion wave will spread out like a fan as it moves through the undisturbed fluid eventually becoming less and less evident. Thus, as depicted in Figure 2, expansion waves spread and disperse as they propagate.

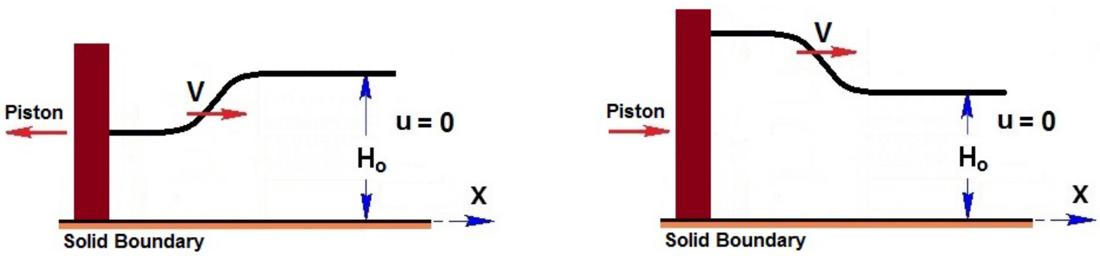

Figure 1: Conceptual formation of an expansion wave (left) and a compression wave (right) in an open channel flow.

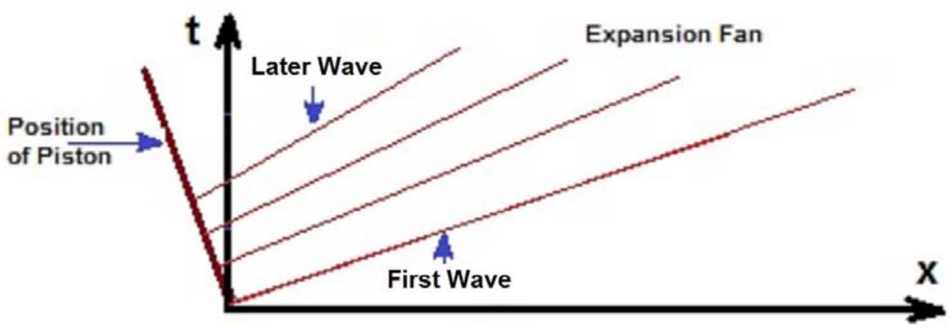

Figure 2: Space/time diagram for an expansion wave in open channel flow.

However, as depicted in Figure 3, a compression wave behaves quite differently. The second perturbation will catch up with the first since it is propagating into deeper fluid where the propagation speed in larger. And the train of perturbations that follow will catch up with their predecessors so as to create a *caustic* and a finite or large amplitude jump. This is variously known as a *hydraulic jump* or *hydraulic bore* depending on the context. It does not readily disperse because a small wave that might separate off and travel ahead of the jump would have a slower speed and thus be absorbed back into the jump. Similarly a small wave that might be left behind in the deeper fluid will travel faster than the jump and thus also be reabsorbed. Waves like this, like a hydraulic jump in open channel flow or a shock wave in compressible flow, are known as waves of *permanent form* since they have this self-preserving character.

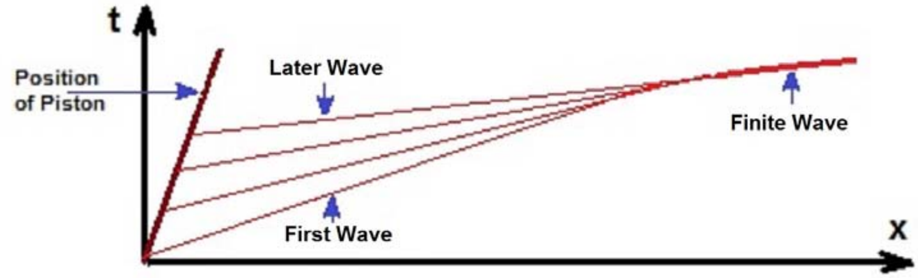

Figure 3: Space/time diagram for a compression wave in open channel flow.

In the next section we provide a detailed analysis of a hydraulic jump or bore and relate the flows upstream and downstream of the jump in the same manner as the Mach numbers upstream and downstream of a gasdynamic shock were related in the section on compressible flow.