Constitutive Relations for a Newtonian Fluid in Cylindrical Coordinates

The constitutive laws for a Newtonian liquid when written in cylindrical coordinates, (r, θ, z), with velocities u_r, u_θ, u_z in the r, θ, z directions become:

$$\sigma_{rr} = -p + 2\mu \frac{\partial u_r}{\partial r} + \Lambda \left\{ \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right\} \quad \text{(Bhd1)}$$

$$\sigma_{\theta\theta} = -p + 2\mu \left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right) + \Lambda \left\{ \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right\} \quad \text{(Bhd2)}$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial u_z}{\partial z} + \Lambda \left\{ \frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z} \right\} \quad \text{(Bhd3)}$$

$$\sigma_{r\theta} = \sigma_{\theta r} = \mu \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \quad \text{(Bhd4)}$$

$$\sigma_{rz} = \sigma_{zr} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \quad \text{(Bhd5)}$$

$$\sigma_{\theta z} = \sigma_{z\theta} = \mu \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) \quad \text{(Bhd6)}$$

where μ is the dynamic viscosity and Λ is the second coefficient of viscosity. For a monatomic gas $\Lambda = -2\mu/3$. The above apply whether the fluid is compressible or incompressible. In the simple case of an *incompressible* fluid the constitutive relations become:

$$\sigma_{rr} = -p + 2\mu \frac{\partial u_r}{\partial r} \quad \text{(Bhd7)}$$

$$\sigma_{\theta\theta} = -p + 2\mu \left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right) \quad \text{(Bhd8)}$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial u_z}{\partial z} \quad \text{(Bhd9)}$$

$$\sigma_{r\theta} = \sigma_{\theta r} = \mu \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r} \right) \quad \text{(Bhd10)}$$

$$\sigma_{rz} = \sigma_{zr} = \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \quad \text{(Bhd11)}$$

$$\sigma_{\theta z} = \sigma_{z\theta} = \mu \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z} \right) \quad \text{(Bhd12)}$$

where μ is the dynamic viscosity.