Choked Flow

Continuing the discussion at the end of the preceding section (Boe), we investigate what happens in subsonic isentropic nozzle flow when the Mach number reaches unity. The general scenario we will consider is a duct that contains a nozzle that ends in a throat that is, in turn, followed by a diffuser as we have depicted in Figure 1. The flow prior to the nozzle will be assumed to be initially subsonic and may or may not have originated in a reservoir though this is not essential to the discussion. Within the converging

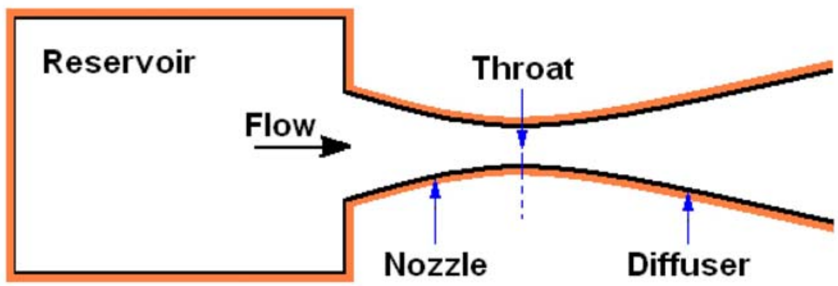

Figure 1: Schematic of general converging/diverging nozzle.

nozzle of Figure 1, the velocity and Mach Number will increase as the cross-sectional area decreases and the pressure, density and temperature will decrease as depicted in Figure 2. So what happens if and when the Mach number reaches unity? To investigate this further we return to the basic governing equations

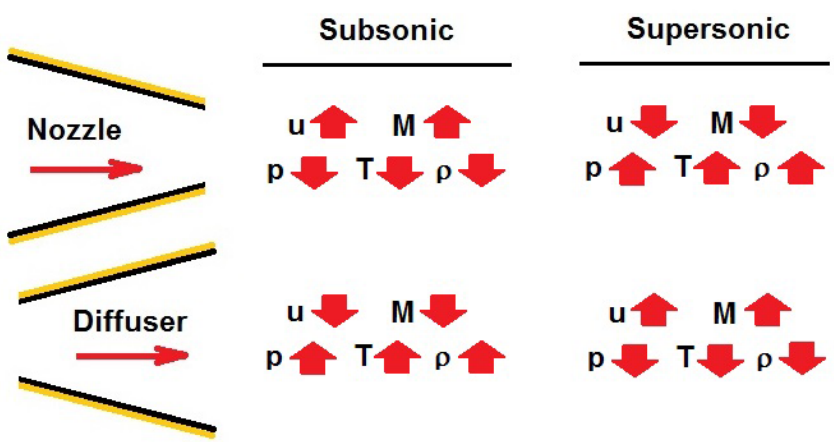

Figure 2: Variations of flow properties in a nozzle and a diffuser.

(Boe1) to (Boe5). By differentiating these equations to obtain the relations between the small changes, dA, du, dM, dp, dT and dρ that will occur in a steady, isentropic duct flow and then eliminating dM, dp, dT and dρ, the following relation between dA and du can be obtained:

$$\frac{dA}{A} \;=\; -\frac{du}{u}\left\{1-M^2\right\}^{\frac{1}{2}} \qquad \text{(Bof1)}$$

and from this we can draw the following conclusion. At a throat where $dA = 0$

1. Either $du = 0$ and $M \neq 1$ so, as long as it remains isentropic, the flow will be subsonic throughout the converging/diverging nozzle because of the trends depicted in Figure 2.

2. Or the Mach number at the throat is unity and du may not be zero. Then there are two sub-options:
   - either du is also zero and the flow remains subsonic as it enters the diffuser and therefore remains subsonic throughout the diffuser, consequently reverting to the first option
   - or $du > 0$ and the flow becomes supersonic in the diffuser and continues supersonic as long as it remains isentropic. We shall see that, somewhere in the diffuser, it may undergo a non-isentropic change by passing through a shock wave becoming subsonic again.

Whichever of the last two sub-options occurs the flow through the converging/diverging nozzle is said to be *choked* for the following reason.

If the flow becomes sonic ($M = 1$) in the throat then, at this location,

$$u^* = c = (\gamma \mathcal{R} T^*)^{\frac{1}{2}} \tag{Bof2}$$

and therefore the mass flow rate through the duct, $\dot{m}$, is given by

$$\dot{m} = \rho^* A^* u^* = \rho^* A^* (\gamma \mathcal{R} T^*)^{\frac{1}{2}} \tag{Bof3}$$

and substituting for ρ^* and T^* from equations (Boe8) this yields

$$\frac{\dot{m}}{A^*} = \rho_0 \left\{ \frac{2}{\gamma+1} \right\}^{\gamma+1/2(\gamma-1)} (\gamma \mathcal{R} T_0)^{\frac{1}{2}} = \gamma^{\frac{1}{2}} \left\{ \frac{2}{\gamma+1} \right\}^{\gamma+1/2(\gamma-1)} \{p_0 \rho_0\}^{\frac{1}{2}} \tag{Bof4}$$

which, for air with $\gamma = 1.4$ becomes,

$$\frac{\dot{m}}{A^*} = 0.685 \{p_0 \rho_0\}^{\frac{1}{2}} \tag{Bof5}$$

In other words the mass flow rate through the duct per unit area of the throat is fixed by the upstream reservoir conditions and does not change irrespective of the downstream conditions. Thus the flow is said to be *choked* in the sense that it does not matter what the downstream conditions are as long as they are such that the Mach number in the throat is unity.

Condsider the following simple and common example shown in Figure 3 that illustrates the phenomenon of choked flow. Gas flows from a reservoir (or source of compressed air) through a nozzle (or even just a

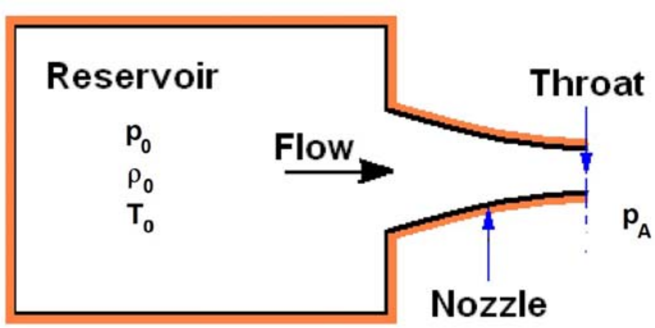

Figure 3: Schematic of a nozzle ejecting to atmosphere.

hole) and out to the atmosphere. For the purposes of the example it is convenient to consider the reservoir to contain gas at constant pressure, temperature and pressure (p_0, T_0 and ρ_0) and to gradually reduce the atmospheric pressure, p_A, from p_0 (at which there would be no flow). The pressure in the throat will be equal to p_A and for small reductions (small $p_0 - p_A$) at which the entire is subsonic the flow out would be small and, if $M \ll 1$ the velocity in the throat and the mass flow rate would be proportional to $(p_0 - p_A)^{\frac{1}{2}}$.

However as p_A is further reduced the Mach number in the throat will eventually approach $M \rightarrow 1$. When it reaches $M = 1$ and the pressure in the throat becomes p^* the flow rate will be given by equation (Bof5) and the flow rate cannot increase further. No matter how much the pressure p_A is reduced the throat pressure will remain at p^* and the flow rate will remain at the value given by equation (Bof5) which depends only on the reservoir pressure and density. The flow is *choked* in that it does not depend on the pressure p_A. The way that the flow downstream of the throat adjusts from p^* in the throat to the external pressure p_A involves complex three-dimensional structures in the flow that will be described in later sections. Finally, we note that this phenomenon of choked flow is often used to control and measure gas flows rates.

By way of a practical example we note from section (Boe) that for $\gamma = 1.4$ (for air) $p^*/p_0 = 0.528$ and therefore a compressed air tank (or an automobile tire) leaking to atmosphere through a hole would exhibit choked flow if the air in the tank had a pressure greater than $1.89atm$ (or about $27.8psi$). As a second example almost any flow through a leak in a pressurized spacecraft or spacesuit would be choked.