Turbomachine Dynamics

In this section we address the one-dimensional model needed to represent the dynamics of an axial flow pump or a propeller in a time-domain treatment of such a component. This will serve as an example for a more general class of machines that inject or extract energy from a flow.

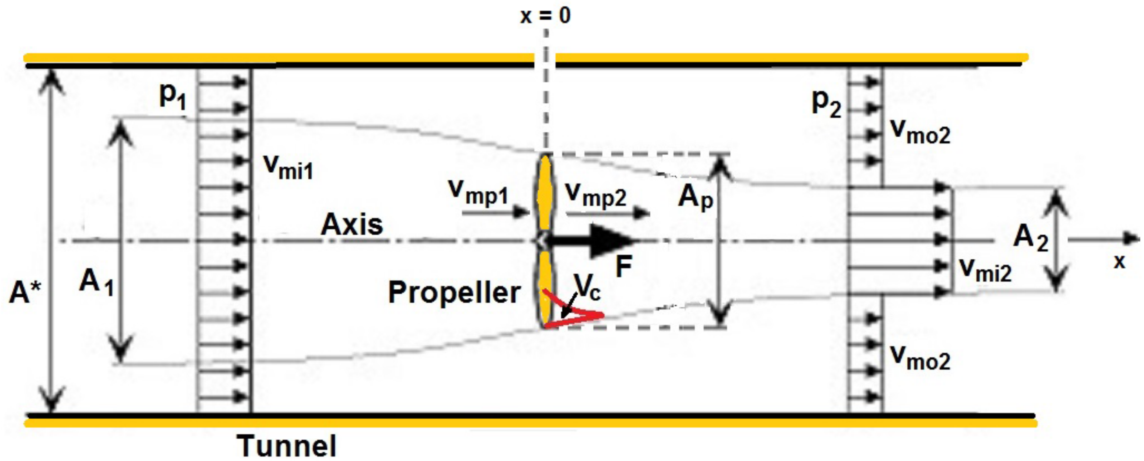

Figure 1: Schematic and notation of an axial flow pump ($A^*/A_p = 1$) or a propeller in a duct or tunnel ($A^*/A_p > 1$); the cavitation volume is shown in red.

We consider the one-dimensional unsteady, incompressible flow through a pump or propeller (either cavitating or non-cavitating) in a tunnel as shown in Figure ??. The impeller or propeller (cross-sectional area A_p) is located on the centerline of the tunnel whose cross-sectional area is A^*. We focus on the stream tube containing the propeller and, for simplicity, it will be assumed that the flow is one-dimensional and uniformly distributed across the propeller stream tube. Friction and mixing losses between the inner and outer flows are neglected. We seek the *unsteady* flow characteristics manifest by such a device both when the propeller is cavitating and when it is not. The mean flow or time-averaged performance of this device was analyzed in Sections (Mfc) and (Mfg) using the basic conservation principles.

To include the unsteady flow contributions, it is necessary to revisit and revise the basic conservation results presented in Sections (Mfc) and (Mfg). The notation used is the same as that used in those sections. Mass conservation requires that

$$v_{mi1}A_1 - v_{mp1}A_p = -\int_{-\infty}^{0} \frac{\partial A(x,t)}{\partial t}\, dx \qquad \text{(Bnfd1)}$$

$$v_{mi2}A_2 - v_{mp2}A_p = \int_{0}^{\infty} \frac{\partial A(x,t)}{\partial t}\, dx \qquad \text{(Bnfd2)}$$

$$v_{mp2}A_p - v_{mp1}A_p = \frac{dV_c}{dt}\, dt \qquad \text{(Bnfd3)}$$

$$v_{mi2}A_2 + v_{mo2}(A^* - A_2) - v_{mi1}A^* = \frac{dV_c}{dt}\, dt \qquad \text{(Bnfd4)}$$

The right-hand-sides of equations (Bnfd1) and (Bnfd2) represent the volume change of the stream tube upstream and downstream of the propeller; later these will be ignored for simplicity. The relation between the pressures far upstream and far downstream is obtained by applying Bernoulli's equation in the outer flow as follows:

$$p_2 - p_2 = \frac{1}{2}\rho\left\{v_{mi1}^2 - v_{mo2}^2\right\} - \rho\int_{-\infty}^{\infty}\frac{\partial v_{mo}(x,t)}{\partial t}\,dx \tag{Bnfd5}$$

where the last term of the right-hand-side is the inertia effect in the control volume.

Now, we calculate the thrust force F produced by the propeller by applying three basic equations. First, applying the momentum theorem to a control volume containing all the tunnel flow, we obtain;

$$\rho v_{mi1}^2 A^* + p_1 A^* + F \;=\; \rho v_{mo2}^2 (A^* - A_2) + \rho v_{mi2}^2 A_2 + p_2 A^* + \frac{dM}{dt} \tag{Bnfd6}$$

The last term in the right-hand-side is rate of the change of the momentum in the control volume, represented by

$$\begin{aligned}\frac{dM}{dt} &= \rho\frac{d}{dt}\left[\int_{-\infty}^{\infty}\left\{v_{mi}(x,t)A(x,t) + v_{mo}(x,t)(A^* - A(x,t))\right\}dx\right]\\ &= \rho\frac{d}{dt}\left[\int_0^{\infty}\frac{dV_c}{dt}\,dx + A^*\int_{-\infty}^{\infty} v_{mi1}dx\right] \;=\; \rho\int_0^{\infty}\frac{d^2V_c}{dt^2}\,dx + \rho A^*\int_{-\infty}^{\infty}\frac{dv_{mi1}}{dt}\,dx\end{aligned} \tag{Bnfd7}$$

which yields

$$\begin{aligned}F &= \frac{1}{2}\rho(v_{mi1} - v_{mo2})A^*(2v_{mi2} + v_{mo2} - v_{mi1}) + \rho(v_{mi2} + v_{mo2})\frac{dV_c}{dt}\\ &\quad + \left[\rho A^*\int_{-\infty}^{\infty}\frac{\partial(v_{mi1} - v_{mo}(x,t))}{\partial t}\,dx + \rho\int_0^{\infty}\frac{d^2V_c}{dt^2}\,dx\right]\end{aligned} \tag{Bnfd8}$$

Second, we obtain the total pressure difference across the propeller, Δp^T, from the Euler head,

$$\Delta p^T \;=\; \rho R\Omega v_{\theta p2} \;=\; \rho R\Omega(R\Omega - v_{mp2}\cot\beta) - \rho\frac{c}{\sin\beta}\frac{dv_{mp2}}{dt} \tag{Bnfd9}$$

The last term in this equation represents the inertia effect of the fluid in the blade passage. Since the static pressure difference, $p_{p2} - p_{p1}$, is given by

$$p_{p2} - p_{p1} \;=\; \frac{1}{2}\rho\left\{R^2\Omega^2 - v_{mp2}^2\cot\beta\right\} - \rho\frac{c}{\sin\beta}\frac{dv_{mp2}}{dt} \tag{Bnfd10}$$

the thrust force can be computed as

$$\begin{aligned}F &= (p_{p2} - p_{p1})A_p + \rho\left\{v_{mp2}^2 - v_{mp1}^2\right\}A_p\\ &= \frac{1}{2}\rho\left\{R^2\Omega^2 - v_{mp2}^2\cot\beta\right\}A_p + \rho(v_{mp2} + v_{mp1})\frac{dV_c}{dt} - \rho\frac{A_p c}{\sin\beta}\frac{dv_{mp2}}{dt}\end{aligned} \tag{Bnfd11}$$

Third, the pressures p_{p1} and p_{p2} may be related to the upstream and downstream conditions using Bernoulli's equation:

$$p_{p1} \;=\; p_1 + \frac{1}{2}\rho v_{mi1}^2 - \frac{1}{2}\rho v_{mp1}^2 - \rho\int_{-\infty}^{0}\frac{\partial v_{mi}(x,t)}{\partial t}\,dx \tag{Bnfd12}$$

where the last term is the inertance in the stream tube. Applying Bernoulli's equation between the outlet of the propeller and far downstream, we obtain

$$p_{p2} \;=\; p_2 + \frac{1}{2}\rho\left[v_{mi2}^2 + v_{\theta p2}^2(A_p/A_2)\right] - \frac{1}{2}\rho\left[v_{mp2}^2 + v_{\theta p2}^2\right] + \rho\int_0^{\infty}\frac{\partial v_{mi}(x,t)}{\partial t}\,dx$$

$$= p_2 + \frac{1}{2}\rho v_{mi2}^2 - \frac{1}{2}\rho v_{mp2}^2 + \frac{1}{2}\rho\left[R\Omega - v_{mp2}\cot\beta\right]^2\left[(A_p/A_2) - 1\right] + \rho\int_0^\infty \frac{\partial v_{mi}(x,t)}{\partial t}\,dx \quad \text{(Bnfd13)}$$

Then the thrust force F follows as

$$\begin{aligned} F &= (p_{p2} - p_{p1})A_p + \rho\left\{v_{mp2}^2 - v_{mp1}^2\right\}A_p \\ &= \frac{1}{2}\rho\left[\left\{v_{mi2}^2 - v_{mo2}^2\right\} + \left\{R\Omega - v_{mp2}\cot\beta\right\}^2\left\{(A_p/A_2) - 1\right\}\right]A_p \\ &\quad -\frac{1}{2}\rho(v_{mp2} + v_{mp1})\frac{dV_c}{dt} + \rho A_p\int_0^\infty \frac{\partial(v_{mi}(x,t) - v_{mo}(x,t))}{\partial t}\,dx \qquad \text{(Bnfd14)} \end{aligned}$$

For the purpose of the general discussion, we have considered all possible unsteady effects in the above formulation, namely the effects of volume change of the stream tubes in equations (Bnfd1) and (Bnfd2), the inertia effects upstream and downstream of the propeller in equations (Bnfd5), (Bnfd8) and (Bnfd14), and the inertia effect in the propeller in equation (Bnfd11) as well as the effects of the cavity volume change dV_c/dt in equations (Bnfd3) and (Bnfd4). To evaluate many of these terms, we would need to know the shape of the stream tube, which is beyond the scope of the present one-dimensional stream tube analysis. Consequently, some compromises are needed in order to proceed. First we neglect the stream tube volume changes in equations (Bnfd1) and (Bnfd2) on the basis that these cancel and thus produce no net perturbation within the water tunnel; this may need further examination. Second, we neglect the inertance terms in equations (Bnfd5), (Bnfd8) and (Bnfd14) on the basis that past experience has suggested that we can consider these contributions to be lumped into the other inertance contributions further upstream and downstream. In the absence of large volumes of cavitation so that $V_c \approx 0$, this completes the model needed for inclusion in a time-domain treatment of a system that incorporates such a device. Note that the eight equations (Bnfd1) through (Bnfd14) contain eight unknowns v_{mo2}, v_{mi2}, v_{mp2}, v_{mp1}, A_1, A_2, F, and p_2 assuming that the propeller operating parameters v_{mi1}, p_1, ΩR, and the discharge flow angle, β, are known. We also note that the empirical relations for the deviation angle that were described in Section (Mfg) will be used again here.

In an unsteady flow in which the propeller is cavitating so that $V_c \neq 0$ and therefore the effects associated with dV_c/dt in equations (Bnfd3) and (Bnfd4) must be included, it is necessary to establish a functional expression for the cavity volume, V_c, and thus complete the set of governing equations. Consistent with the understanding developed in the context of cavitating pumps it will be assumed that cavity volume, $V_c(p_{p1}, v_{mp1})$, is a function of the inlet pressure p_{p1} and inflow velocity v_{mp1}. Then, the rate of change of the cavity volume can be expressed as

$$\frac{dV_c}{dt} = -K\frac{dp_{p1}}{dt} - M\frac{dv_{mp1}}{dt} \qquad \text{(Bnfd15)}$$

where $K = -\partial V_c/\partial p_{p1}$ and $M = -\partial V_c/\partial v_{mp1}$ are respectively the cavitation compliance and the mass flow gain factor (see Sections ???? and Brennen and Acosta 1973). These important parameters are non-dimensionalized as follows;

$$\frac{K^*}{2\pi} = -\frac{\partial(V_c/A_pR)}{\partial\sigma^*} = \frac{\rho R\Omega^2}{2A_p}\frac{\partial V_c}{\partial p_{p1}} = \frac{\rho\Omega^2}{2\pi R}K \qquad \text{(Bnfd16)}$$

$$M^* = -\frac{\partial(V_c/A_pR)}{\partial(v_{mp1}/R\Omega)} = \frac{\Omega}{A_p}\frac{\partial V_c}{\partial v_{mp1}} = \frac{\Omega}{\pi R^2}M \qquad \text{(Bnfd17)}$$

where K^* and M^* are the non-dimensional values of the cavitation compliance and the mass flow gain factor used by Duttweiler and Brennen (2002). Typical values for K and M are estimated by Otsuka *et al.* (1996) and Watanabe *et al.* (1998) using free streamline theory. Here, we utilize their results in order to estimate appropriate values of $K^*/2\pi$ and M^*. The values of $(K^*/2\pi,\ M^*)$ obtained by those investigations are shown in Figure **??** for typical values for the solidity (1.0), the stagger angle ($\beta = 25°$) and the number of blades ($Z_R = 5$). Because Otsuka *et al.* (1996) and Watanabe *et al.* (1998) examine only two-dimensional flows around foils, the cavity size per blade is treated as a cross sectional area V_{cpb} (not a volume) and the scaling as $V_c = Z_R R V_{cpb}/2$ is used as a best estimate. Note that $(K^*/2\pi,\ M^*)$ are functions of the parameter $\lambda = \sigma^*/2\alpha$, where σ^* is the cavitation number *at inlet to the propeller.*

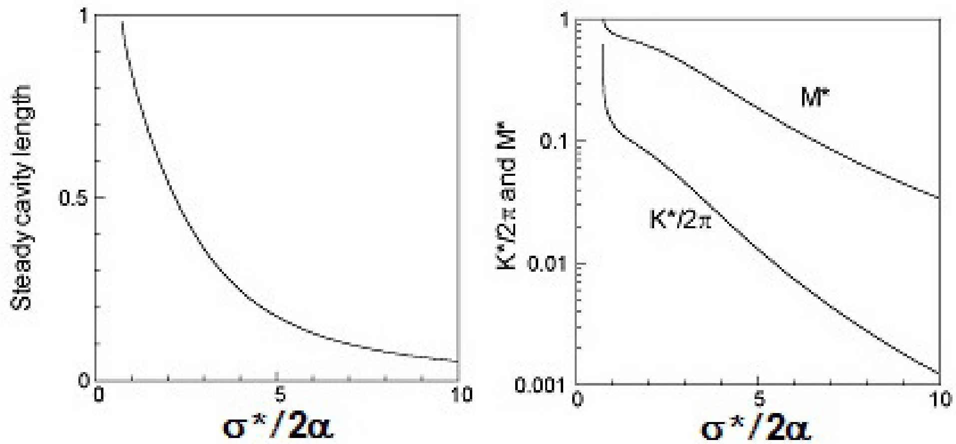

Figure 2: Steady cavity length and the quasi-static cavitation compliance and mass flow gain factor plotted against $\sigma^*/2\alpha$ obtained by a free streamline theory (Watanabe *et al.* 1998) for solidity = 1.0, stagger angle $\beta = 25°$ and $Z_R = 5$.

The cavitation compliance $K^*/2\pi$ varies from 0.018 to 0.172 for $A^*/A_p = 2$ and from 0.009 to 0.143 for $A^*/A_p = 10$. The mass flow gain factor M^* varies from 0.231 to 0.831 for $A^*/A_p = 2$ and from 0.140 to 0.777 for $A^*/A_p = 10$. These values are slightly smaller for the case with $A^*/A_p = 10$. In the dynamics of other similar devices, the mass flow gain factor plays an important role in promoting instability. It is therefore noteworthy that the largest mass flow gain factors occur for the smaller values of $\sigma^*/2\alpha$ and that this occurs when the mean cavity length approaches the chord length. This accords with the observation that instability can set in when the cavity length approached the chord of the propeller (Duttweiler and Brennen 2002).