Turbomachines

Turbomachines for incompressible flows, namely pumps and turbines, are ubiqitous components in hydraulic systems and their design at the most basic level is governed by a set of relations that are known as the **affinity laws**. These are relations derived by dimensional analysis based on the fundamental features of the turbomachine namely the density of the fluid, ρ, the typical rotational speed, Ω, and the typical diameter, D, of the machine. It follows from dimensional analysis that the performance characteristics of the machine, namely the volume flow rate through the machine, Q, the total head change across the machine, H, the shaft torque, T, and the power transmitted to or from the machine, P, will scale according to

$$\begin{aligned} Q &\quad \alpha \quad \Omega D^3 \\ H &\quad \alpha \quad \Omega^2 D^2 \\ T &\quad \alpha \quad \rho D^5 \Omega^2 \\ P &\quad \alpha \quad \rho D^5 \Omega^3 \end{aligned} \tag{Bfg1}$$

These simple relations allow basic scaling predictions and initial design estimates. Furthermore, they permit consideration of optimal characteristics, such as the power density which, according to the above, should scale like $\rho D^2 \Omega^3$. Though the constraints on a turbomachine design are as varied as the almost innumerable applications, there are a number of ubiquitous trends which allow some fairly general conclusions to be drawn.

One typical consideration arising out of the affinity laws relates to optimizing the design of a pump for a particular power level, P, and a particular fluid, ρ. This fixes the value of $D^5\Omega^3$. If one wished to make the pump as small as possible (small D) to reduce weight (as is critical in the rocket engine context) or to reduce cost, this would dictate not only a higher rotational speed, Ω, but also a higher impeller tip speed, $\Omega D/2$. However, increasing the speed can lead to a number of critical problems, particularly in a liquid turbomachine. We will focus here on two of those problems caused respectively by the possibility that might vaporize and by the forces caused by the high liquid density.

Addressing the first of these problems, the phenomenon of the formation of vapor bubbles in regions of low pressure within the flow field of a liquid is called **cavitation**. In some respects, cavitation is similar to boiling, except that the latter is generally considered to occur as a result of an increase of temperature rather than a decrease of pressure. This difference in the direction of the state change in the phase diagram is more significant than might, at first sight, be imagined. It is virtually impossible to cause any rapid uniform change in temperature throughout a finite volume of liquid. Rather, temperature change most often occurs by heat transfer through a solid boundary. Hence, the details of the boiling process generally embrace the detailed interaction of vapor bubbles with a solid surface, and the thermal boundary layer on that surface. On the other hand, a rapid, uniform change in pressure in a liquid is commonplace and, therefore, the details of the cavitation process may differ considerably from those that occur in boiling. Much more detail on the process of cavitation is included in other sections of this book.

Cavitation is generally a malevolent process, and the deleterious consequences that it causes can be divided into three categories. First, cavitation can cause damage to the material surfaces close to the area where the bubbles collapse when they are convected into regions of higher pressure. Cavitation damage can be very expensive, and very difficult to eliminate. For most designers of hydraulic machinery, it is the preeminent problem associated with cavitation. The second adverse effect of cavitation is that the performance of the pump, or other hydraulic device, may be significantly degraded. The third adverse effect of cavitation is less well known, and is a consequence of the fact that cavitation affects not only the steady state fluid flow, but also the unsteady or dynamic response of the flow. This change in the dynamic performance leads to instabilities in the flow that do not occur in the absence of cavitation. For present purposes it is sufficient to note that the propensity for cavitation increases as a parameter called the **cavitation number** decreases, and the cavitation number is inversely proportional to the square of the tip speed of the turbomachine, $\Omega^2 D^2/4$. Consequently, the increase in speed suggested above could lead to a cavitation problem. Often, therefore, one designs the smallest turbomachine that will still operate without cavitation, and this implies a particular size and speed for the device.

The other major class of problems encountered as the speed is increased are those generated by the larger fluid forces and these tend to be much greater in a liquid turbomachine because of the larger density. Typically, fluid dynamic forces scale like $\rho\Omega^2 D^4$ where ρ is the fluid density, and Ω and D are the typical frequency of rotation and the typical length, such as the span or chord of the impeller blades or the diameter of the impeller. These forces are applied to blades whose typical thickness is denoted by τ. It follows that the typical structural stresses in the blades are given by $\rho\Omega^2 D^4/\tau^2$, and, if $D^5\Omega^3$ is fixed and if one maintains the same geometry, D/τ, then the stresses will increase like $D^{-4/3}$ as the size, D, is decreased. Consequently, fluid/structure interaction problems will increase. To counteract this the blades are often made thicker (D/τ is decreased), but this usually leads to a decrease in the hydraulic performance of the turbomachine. Consequently an optimal design often requires a balanced compromise between hydraulic and structural requirements. Rarely does one encounter a design in which this compromise is optimal.

Of course, the design of a pump, compressor or turbine involves many factors other than the technical issues discussed above. Many compromises and engineering judgments must be made based on constraints such as cost, reliability and the expected life of a machine.