RADIAL FLOW

Purely radial flow also contributes to the exact solutions of the Navier-Stokes equations. Though planar radial flow from a line source or an expanding cylinder could be counted here, the present focus will be

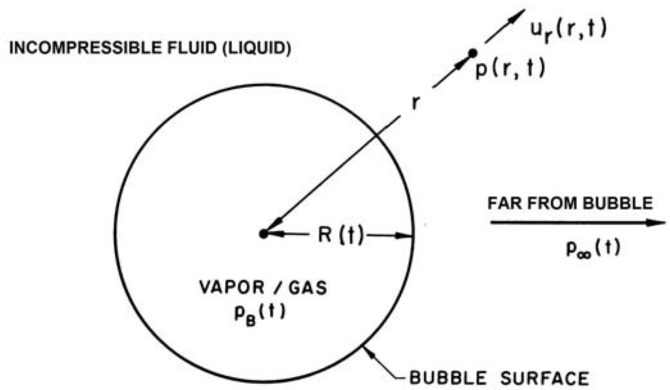

Figure 1: Spherically symmetric radial flow.

on spherically symmetric radial flow such as that produced by a point source or an expanding spherical bubble as depicted in Figure 1. In all these flows the only non-zero component of velocity is the radial velocity, u_r. The continuity equation for such an incompressible radial flow simply requires that

$$u_r \; = \; \frac{F(t)}{r^2} \tag{Bid1}$$

where $F(t)$ could be any function of time including a constant. The Navier-Stokes equations in spherical coordinates for a incompressible fluid with uniform and constant viscosity (see section (Bhh)) but with all velocities except u_r set to zero and all derivatives in the θ and ϕ directions also set to zero produce just one consequential equation, namely

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} - \nu\left\{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial u_r}{\partial r}) - \frac{2u_r}{r^2}\right\} \tag{Bid2}$$

where ρ and ν are the density and kinematic viscosity of the fluid. After substituting for u_r from equation (Bid1), equation (Bid2) yields

$$-\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{1}{r^2}\frac{dF}{dt} - \frac{2F^2}{r^5} \tag{Bid3}$$

Note that the viscous terms vanish; indeed, the only viscous contribution to the fluid dynamics comes from the dynamic boundary condition at the interior surface at $r = R$. Equation (Bid3) can be integrated to give

$$\frac{p - p_\infty}{\rho} = \frac{1}{r}\frac{dF}{dt} - \frac{1}{2}\frac{F^2}{r^4} \tag{Bid4}$$

after application of the condition $p \rightarrow p_\infty$ as $r \rightarrow \infty$.

If there is an interior Lagrangian spherical surface at radius $r = R(t)$ at which the pressure is known and denoted by $p_B(t)$ then it follows that

$$F(t) = R^2\frac{dR}{dt} \tag{Bid5}$$

then substituting this into equation (Bid4) yields a relation between the pressure, $p_B(t)$, and the position of that surface, $R(t)$:

$$\frac{p_B(t) - p_\infty(t)}{\rho} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 \tag{Bid6}$$

Given $p_\infty(t)$ and $p_B(t)$this represents an equation that can be solved to find $R(t)$. This equation was first put forward by Lord Rayleigh in his study of the dynamics of spherical bubbles. It forms the background of the more extensive modeling of cavitation bubbles that is detailed in section (Ngb) and, with additional terms due to surface tension and viscous effects in the bubble surface condition, is now widely known and used as the Rayleigh-Plesset equation.