Supersonic Potential Flow

The solutions for supersonic potential flow around slender bodies have already been covered in section (Bok) but some features are valuable to emphasize in the context of the present discussion. For simplicity we illustrate these features in planar flows though most of the comments also apply to general, three-dimensional flows. The hyperbolic partial differential equation that governs such a planar flow (see equation (Bon9)) is

$$(M^2-1)\frac{\partial^2\phi}{\partial x_1^2} = \frac{\partial^2\phi}{\partial x_2^2} \qquad \text{(Boo1)}$$

and this wave equation has general solutions of the form

$$\phi = Ux_1 + f\left(x_1 \mp (M^2-1)^{\frac{1}{2}}\, x_2\right) \qquad \text{(Boo2)}$$

Consequently all flow properties are constant along the characteristic lines

$$x_1 \mp (M^2-1)^{\frac{1}{2}}\, x_2 = \text{constant} \qquad \text{(Boo3)}$$

and these, of course, include the Mach waves described in section (Bok). In practical application only the waves beginning at a disturbance such as that generate by a solid surface and proceeding downstream will propagate significant perturbations. It is convenient to define the characteristic coordinate

$$\xi = x_1 \mp (M^2-1)^{\frac{1}{2}}\, x_2 \qquad \text{(Boo4)}$$

where the upper sign is used for the upper, suction surface of the foil in Figure 1 and the lower sign is used for the lower, pressure surface. It follows that

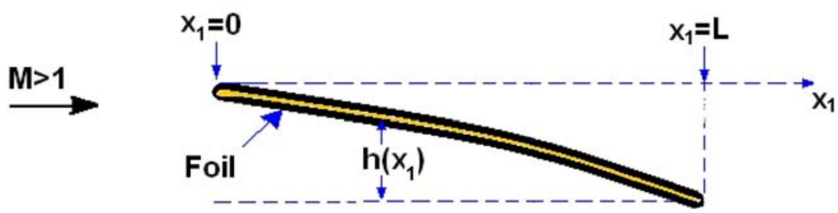

Figure 1: Supersonic potential flow past a thin, curved airfoil.

$$u_1 = U + \frac{df}{d\xi} \qquad \text{and} \qquad u_2 = \mp(M^2-1)^{\frac{1}{2}}\,\frac{df}{d\xi} \qquad \text{(Boo5)}$$

For these slender body flows the approximate boundary condition on the surface, equation (Bon11), yields

$$u_2 = U\frac{dh}{dx_1} = \mp(M^2-1)^{\frac{1}{2}}\,\frac{df}{d\xi} \qquad \text{(Boo6)}$$

so that

$$\frac{df}{d\xi} = \mp\frac{U}{(M^2-1)^{\frac{1}{2}}}\frac{dh}{dx_1} \qquad \text{(Boo7)}$$

and therefore from equations (Boo5)

$$u_1 = U \mp \frac{U}{(M^2-1)^{\frac{1}{2}}}\frac{dh}{dx_1} \qquad \text{(Boo8)}$$

and therefore from equation (Bon14), the coefficients of pressure, C_p, on the two surfaces are

$$C_p = \pm \frac{2}{(M^2-1)^{\frac{1}{2}}} \frac{dh}{dx_1} \tag{Boo9}$$

and the lift coefficient, C_L, from equation (Bon15) becomes

$$C_L = -\frac{1}{L}\int_0^L \frac{4}{(M^2-1)^{\frac{1}{2}}} \frac{dh}{dx_1}\, dx = -\frac{4}{L(M^2-1)^{\frac{1}{2}}} \{h(L)-h(0)\} \tag{Boo10}$$

and, if the angle of attack, α, is defined as $(h(0)-h(L))/L$ then we recover the expected result (see section (Bok)) that

$$C_L = \frac{4\alpha}{(M^2-1)^{\frac{1}{2}}} \tag{Boo11}$$

In addition, the drag coefficient, C_D, is given by

$$C_D = \frac{4}{L(M^2-1)^{\frac{1}{2}}} \int_0^L \left(\frac{dh}{dx}\right)^2 dx \tag{Boo12}$$