Numerical Methods

Computational methods now play a large part in both fluids engineering and in fluid mechanics research. The methods used are too numerous and diverse to cover adequately in this text. Rather we shall try to provide a short introduction designed for the student who is new to the topic and we will provide an example that is intended as a learning aid.

There are two different philosophical starting points in the effort to create a numerical solution for a particular flow (or for that matter a particular stress/strain state in a solid). The first begins with a set of differential equations and boundary conditions governing the fluid motion and proceeds to develop ways to obtain approximate numerical solutions to those equations in the form of values for the fluid velocities, pressures and other flow and state vatiables at a set of points in the flow field and at its boundaries. Such efforts are known as **finite difference methods**. The second approach divides the entire flow field into a mesh of finite elements and constructs an appropriate set of conservation principles (conservation of mass, momentum, energy, etc.) that each and every one of these elements must satisfy. Each element has associated with it a set of flow variable values at various location within or on the boundary of the element and the first task is to find an expression for each of those conservation principles in terms of the associated numerical values. Such methods are known as **finite element methods**. Both approaches strive to obtain approximate values of the fundamental flow variables at a set of points in the flow field and at its boundaries. Once those values have been obtained, by whichever method, there is usually a post-solution effort in which the engineer or researcher strives to understand the solution that has been obtained and the effect that the imposed initial conditions or boundary conditions have had on the flow. A frequent part of that effort involves visualizing the flow so much effort is frequently devoted to such visualizations [often these visualizations are used primarily for sales purposes].

An expanding use of numerical solutions and their post-processing visualizations and analyses is in conjunction with experiments and tests, both of the research type and of the product development type. The engineer or researcher sets up a CFD processor to access the principal variables of the experiment on line and to produce a CDF solution to that flow as the experiment is running thus allowing direct and immediate comparison between the experimental results and the computational results. Such an aid to the experiments/tests allows the researcher/investigator to explore the observed phenomena and interrogate the experiment/test in a greatly enhanced way while also exploring ways to improve the computational simulation.

In this text we will confine attention to some of the simplest and most rudimentary methods of finite differences and finite elements. A prerequisite for both approaches is to fill the space occupied by the flow with a mesh or grid. While the two different approaches may have different grid preferences that differ, there are some similar requirements. For example in both approaches, there are regions of the flow where the gradients of the pertinent flow variables are much larger than elsewhere. Clearly, the mesh or grid needs to be much finer in those regions in order to accurately resolve those gradients. At the other end of the scale, there are regions in the flow far away where we may wish to stipulate that the flow approaches a simple form, such as that of a uniform stream. Often, therefore, the researcher wishes to patch that simple flow type (usually with some adjustable constants) into the main computational domain and the ways in which this is done are similar in the two fundamental approaches. Indeed, considerable effort is often expended in the design of a mesh before any fluid flow equations or principles are considered. Thus mesh design has become a subject of research in itself.

All of the preceding comments apply to computational methods for any type of flow. However, the details of the methods can vary greatly with the type of flow under investigation. In this regard we should identify several major categories. One essential distinction is between those flows governed by an elliptical partial differential equations and those governed by hyperbolic partial differential equations (for example, between subsonic flows and supersonic flows or between subcritical open channel flows and supercritical open channel flows). Another distinction is between steady flows and unsteady flows. Where appropriate we will comment on these in the sections devoted to that particular kind of flow. For the purposes of the present example, we choose to focus on simple potential flows which are governed by an elliptic differential equation and to indicate some of the details of the methodology without trying to cover the variations in other types of flow.