Basic Fluid Mechanics of Turbine Flows

Further evolution in the design of turbines came with the experience that some geometrical designs worked better in some operational conditions and some in others as described in sections (Mac). The fluid mechanics of this spectrum of turbines are similar to that of rotordynamic pumps described in sections (Mbbf) and (Mbbg). However, the differences make it valuable to repeat that analysis here. Both axial and cen-

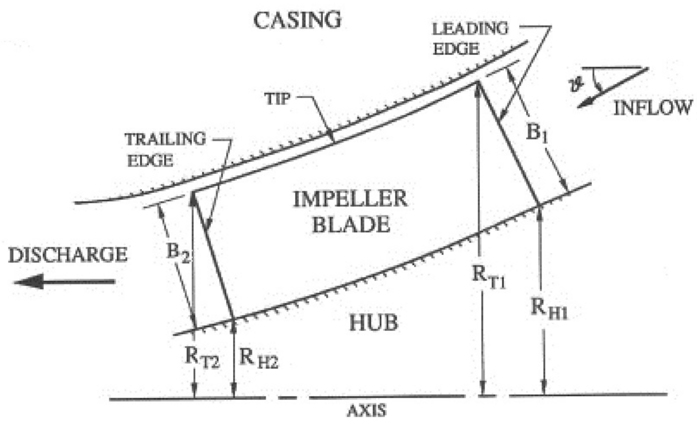

Figure 1: Cross-sectional view through the axis of a turbine runner.

trifugal turbines (propeller, Kaplan and Francis turbines) can, initially, be treated using the same notation and analysis. That common geometric model and notation is sketched in Figure 1, and consists of a set of runner (rotor) blades (number = Z_R) attached to a hub and operating within a static casing. The radii of the inlet blade tip, inlet blade hub, discharge blade tip, and discharge blade hub are denoted by R_{T1}, R_{H1}, R_{T2}, and R_{H2}, respectively. The inlet flow to the runner is inclined to the axis of rotation at an angle, ϑ, which would be close to 90° in the case of a Francis turbine and small in the case of a propeller or Kaplan machine. The inlet flow usually proceeds through a volute, through stationary vanes and/or through wicket gates so that it approaches the runner with a substantial swirl velocity, $v_{\theta 1}$. The swirl angle of that flow, $\beta_i(r)$, is defined as the inclination of the fluid velocity vector approaching the runner in the meridional plane to the plane perpendicular to the axis of rotation as depicted in Figure 2. It follows from the velocity triangles of Figure 2 that

$$\cot\beta = \cot\beta_i - \frac{\Omega r}{v_m} \quad \text{(Mde1)}$$

In the present analysis the turbine performance will be evaluated assuming that $\beta_i(r)$ is known. Then if $\Omega r/v_m$ is given, the inlet flow angle, $\beta(r)$, can be determined (see below for further comment).

The flow through the runner is normally visualized by developing a meridional surface that can either correspond to an axisymmetric stream surface, or be some estimate thereof. On this meridional surface (see figure 2) the fluid velocity in a non-rotating coordinate system is denoted by $v(r)$ (with subscripts 1 and 2 denoting particular values at inlet and discharge) and the corresponding velocity relative to the rotating blades is denoted by $w(r)$. The velocity, v, has components v_θ in the circumferential direction, and v_m in the meridional direction. The velocity of the blades is Ωr. As shown in figure 2, the flow angle $\beta(r)$ is defined as the angle between the relative velocity vector in the meridional plane and a plane perpendicular to the axis of rotation. The blade angle $\beta_b(r)$ is defined as the inclination of the tangent to the blade in the meridional plane and the plane perpendicular to the axis of rotation. If the flow is precisely parallel

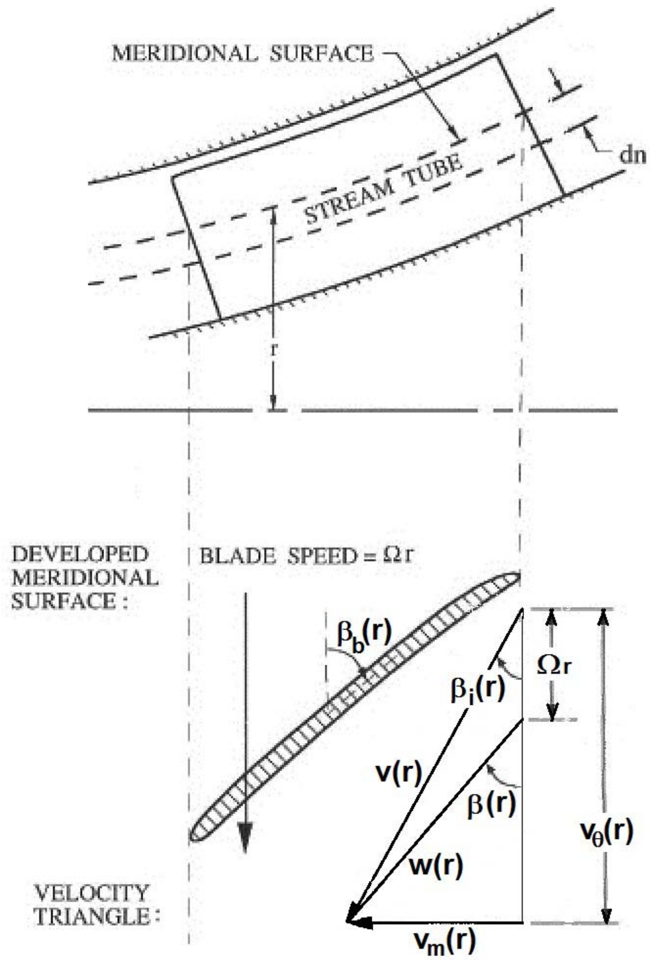

Figure 2: Developed meridional surface and velocity triangle.

to the blades, $\beta = \beta_b$. Specific values of the blade angle at the leading and trailing edges (1 and 2) and at the hub and tip (H and T) are denoted by the corresponding suffices, so that, for example, β_{bT2} is the blade angle at the discharge tip.

At the leading edge of the runner it is important to know the angle $\alpha(r)$ with which the flow meets the blades of the runner. This angle is controlled by the angle at which the flow discharges from the wicket gates and that angle, in turn, depends on the inclination of the wicket gates themselves. As described previously the wicket gate inclination is a key element in the determining the torque generated by the runner and therefore, in many installations, it is critical to the control of the output and speed of the turbine. As depicted in figure 3, the angle $\alpha(r)$ with which the flow meets the blades of the runner is given by

$$\alpha(r) = \beta_{b1}(r) - \beta_1(r). \tag{Mde2}$$

and is called the incidence angle. For simplicity, we denote the values of the incidence angle at the tip, $\alpha(R_{T1})$, and at the hub, $\alpha(R_{H1})$, by α_T and α_H, respectively.

The incidence angle for the runner should not be confused with the "angle of attack", which is the angle between the incoming relative flow direction and the chord line (the line joining the leading edge to the trailing edge). Note, however, that, in an axial flow turbine with straight helicoidal blades, the angle of attack is equal to the incidence angle.

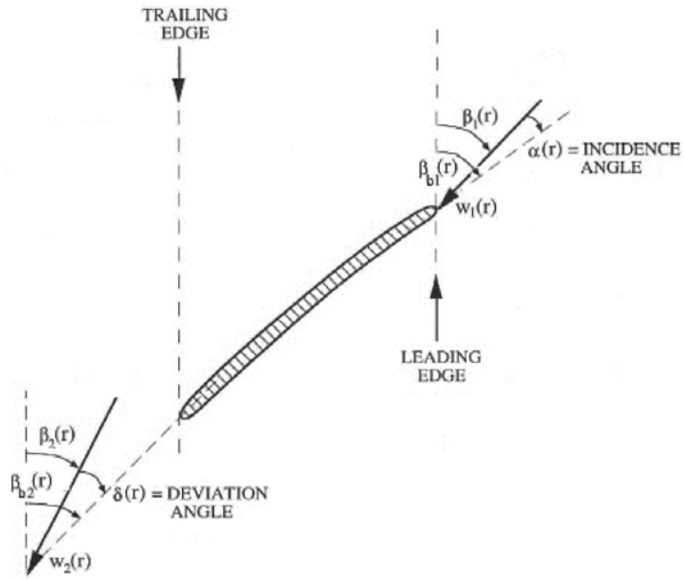

Figure 3: Repeat of figure 2 showing the definitions of the incidence angle at the leading edge and the deviation angle at the trailing edge.

At the trailing edge, the difference between the flow angle and the blade angle is again important. To a first approximation one often assumes that the flow is parallel to the blades, so that $\beta_2(r) = \beta_{b2}(r)$. A departure from this idealistic assumption is denoted by the deviation angle, $\delta(r)$, where, as shown in figure 3:

$$\delta(r) = \beta_{b2}(r) - \beta_2(r) \tag{Mde3}$$

This is normally a function of the ratio of the width of the passage between the blades to the length of the same passage, a geometric parameter known as the solidity that is defined more precisely in section (Mbbb). Other angles that are often used are the angle through which the flow is turned, known as the *deflection angle*, $\beta_2 - \beta_1$, and the corresponding angle through which the blades have turned, known as the *camber angle* and denoted by $\theta_c = \beta_{b2} - \beta_{b1}$. Deviation angles in radial machines are traditionally represented by the *slip velocity*, $v_{\theta s}$, which is the difference between the actual and ideal circumferential velocities of the discharge flow.

The flow variables that are important are, of course, the static pressure, p, the total pressure, p^T, and the volume flow rate, Q. Often the total pressure is defined by the total head, $p^T/\rho g$. Moreover, in most situations of interest in the context of turbomachinery, the potential energy associated with the earth's gravitational field is negligible relative to the kinetic energy of the flow, so that, by definition

$$p^T = p + \frac{1}{2}\rho v^2 \tag{Mde4}$$

$$p^T = p + \frac{1}{2}\rho\left(v_m^2 + v_\theta^2\right) \tag{Mde5}$$

$$p^T = p + \frac{1}{2}\rho\left(w^2 + 2r\Omega v_\theta - \Omega^2 r^2\right) \tag{Mde6}$$

using the velocity triangle of figure 2. In an incompressible flow, the total pressure represents the total mechanical energy per unit volume of fluid, and, therefore, the loss of total pressure across the turbine, $p_1^T - p_2^T$, is a fundamental measure of the mechanical energy imparted by the fluid to the runner.

It follows that, in a turbine with an incompressible fluid, the overall characteristics that are important are the volume flow rate, Q, the total pressure drop, $\rho g H$ (where $H = (p_1^T - p_2^T)/\rho g$ is the total head drop), and the torque, T, and power, P, transmitted to the runner ($P = T\Omega$). As in the case of a pump, it is convenient to present these characteristics non-dimensionally as follows.

The total pressure drop, $(p_1^T - p_2^T) = \rho g H$, can be defined by a non-dimensional head coefficient,

$$\psi = (p_1^T - p_2^T)/\rho R_{T1}^2 \Omega^2 = gH/R_{T1}^2 \Omega^2 \quad \text{(Mde7)}$$

which is identical to that employed in the pump context. However, a different coefficient, ψ^*, based on the head and the runner velocity has traditionally been used in the presentation of turbine test data. This non-dimensional number is based on a fluid velocity, $(2gH)^{\frac{1}{2}}$, derived from the head, H, and sometimes termed the "spouting velocity". Then

$$\psi^* = \Omega R_{T1}/(2gH)^{\frac{1}{2}} = (2\psi)^{-\frac{1}{2}} \quad \text{(Mde9)}$$

[*Note that the turbine literature often uses the symbol ϕ instead ψ^* but this has been changed here to avoid confusion with the flow coefficient and to be consistent with the pump nomenclature.*]

Various definitions are used for the flow coefficient, ϕ. Here we use

$$\phi = Q/R_{T1}^3 \Omega \quad \text{or} \quad \phi^* = Q/R_{T1}^2 (2gH)^{\frac{1}{2}} = \phi/(2\psi)^{\frac{1}{2}} \quad \text{(Mde8)}$$

where the first uses the characteristic velocity $R_{T1}\Omega$ while the second uses the characteristic velocity $(2gH)^{\frac{1}{2}}$. The former differs only in detail from that used in the pump context. In addition a torque (or power coefficient), $\mathcal{T}$, is useful where

$$\mathcal{T} = T/\rho R_{T1}^5 \Omega^2 = P/\rho R_{T1}^5 \Omega^3 \quad \text{(Mde10)}$$

Frequently, the conditions at inlet to and/or discharge from the runner are nonuniform and one must subdivide the flow into annular stream tubes, as indicated in figure 2. Each stream tube must then be analyzed separately, using the blade geometry pertinent at that radius. The mass flow rate, m, through an individual stream tube is given by

$$m = 2\pi \rho r v_m dn \quad \text{(Mde11)}$$

where n is a coordinate measured normal to the meridional surface, and, in the present text, will be useful in describing the discharge geometry.

Conservation of mass requires that m have the same value at inlet and discharge. This yields a relation between the inlet and discharge meridional velocities, that involves the cross-sectional areas of the stream tube at these two locations. The total volume flow rate through the turbine, Q, is then related to the velocity distribution at any location by the integral

$$Q = \int 2\pi r v_m(r) dn \quad \text{(Mde12)}$$

The total head drop across the turbine, H, is given by the integral of the total rate of work done by the flow on the runner divided by the total mass flow rate:

$$H = \frac{1}{Q} \int \frac{(p_1^T(r) - p_2^T(r))}{\rho g} 2\pi r v_m(r) dn \quad \text{(Mde13)}$$

These integral expressions for the flow rate and head rise are useful in more detailed analyses of the flow through the turbine. The next step in the assessment of the performance of a turbine is to consider the application of the first and second laws of thermodynamics. In doing so we shall characterize the inlet and discharge flows by their pressure, velocity, enthalpy, etc., assuming that these are uniform flows. It is understood that when the inlet and discharge flows are non-uniform, the analysis actually applies to a single stream tube and the complete energy balance requires integration over all of the stream tubes.

The basic thermodynamic measure of the energy stored in a unit mass of flowing fluid is the total specific enthalpy (total enthalpy per unit mass) denoted by h^T and defined by

$$h^T = h + \frac{1}{2}|u|^2 + gz = e + \frac{p}{\rho} + \frac{1}{2}|u|^2 + gz \qquad \text{(Mde14)}$$

where e is the specific internal energy, $|u|$ is the magnitude of the fluid velocity, and z is the vertical elevation. This expression omits any energy associated with additional external forces (for example, those due to a magnetic field), and assumes that the process is chemically inert.

Consider the steady state operation of a turbine in which the entering fluid has a total specific enthalpy of h_1^T, the discharging fluid has a total specific enthalpy of h_2^T, the mass flow rate is ρQ, the net rate of heat addition to the machine is $\mathcal{Q}$, and the net rate of work done by the fluid on the runner by external means is P. It follows from the first law of thermodynamics that

$$\rho Q(h_2^T - h_1^T) = \mathcal{Q} - P \qquad \text{(Mde15)}$$

Now consider incompressible, inviscid flow. It is a fundamental property of such a flow that it contains no mechanism for an exchange of thermal and mechanical energy, and, therefore, equation (Mde15) divides into two parts, governing the mechanical and thermal components of the total enthalpy, as follows

$$(p/\rho + \frac{1}{2}|u|^2 + gz)_1 - (p/\rho + \frac{1}{2}|u|^2 + gz)_2 = \frac{(p_1^T - p_2^T)}{\rho} = \frac{P}{\rho Q} \qquad \text{(Mde16)}$$

$$e_2 - e_1 = \mathcal{Q}/\rho Q \qquad \text{(Mde17)}$$

Thus, for incompressible inviscid flow, the fluid mechanical problem (for which equation (Mde16) represents the basic energy balance) can be decoupled from the heat transfer problem (for which the heat balance is represented by equation (Mde17)).

It follows that, if T is the torque applied by the fluid to the runner, then the rate of work done by the fluid is $P = T\Omega$. Consequently, in the case of an ideal fluid that is incompressible and inviscid, equation (Mde15) yields a relation connecting the total pressure drop across the turbine, $p_1^T - p_2^T$, the mass flow rate, ρQ, and the torque:

$$Q(p_1^T - p_2^T) = T\Omega \qquad \text{(Mde18)}$$

Furthermore, the second law of thermodynamics implies that, in the presence of irreversible effects such as those caused by viscosity, the equality in equation (Mde18) should be replaced by an inequality, namely a "greater than" sign. Consequently, in a real turbine operating with an incompressible fluid, viscous effects will cause some of the energy in the inflow to be converted to heat rather than to work done on the runner. It is, therefore, appropriate to define a quantity, η_T, known as the turbine hydraulic efficiency to represent that fraction of energy in the incoming flow that ends up as mechanical energy imparted to the runner:

$$\eta_T = \frac{T\Omega}{Q\,(p_1^T - p_2^T)} = \frac{\mathcal{T}}{\psi\phi} \qquad \text{(Mde19)}$$

in terms of the non-dimensional coefficients defined in equations (Mde7) through (Mde10).

Of course, additional mechanical losses may occur in a turbine. These can cause the rate of work transmitted out of the turbine through the shaft to be greater than the rate of work transmitted to the runner. For example, losses may occur in the bearings or as a result of the "disk friction" losses caused by the fluid dynamic drag on other, non-active surfaces rotating with the shaft. Consequently, the overall turbine efficiency, η_S, may be significantly smaller than η_T. Despite all these loss mechanisms, turbines, especially large ones, can be remarkably efficient. A well designed hydraulic turbine should have an overall efficiency greater than 85% and some very large turbines can exceed 90%.

In addition to the relation (Mde19), just as in the pump analysis of section (Mbbg), the angular momentum theorem applied to a streamtube within the runner relates the incremental torque, dT, to the net flux of angular momentum into the streamtube runner or

$$dT = m(r_1 v_{\theta 1} - r_2 v_{\theta 2}) \tag{Mde20}$$

which holds even when there are viscous losses.

The fluid mechanical analyses continue in the next section in which we explore approximate two-dimensional forms of the above equations.