## An Internet Book on Fluid Dynamics

## Annulus at High Reynolds Numbers

Consider now the flows of the last section when the Reynolds numbers become much greater than unity. The name "bearing" must be omitted, since the flow no longer has the necessary rotordynamic characteristics to act as a hydrodynamic bearing. Nevertheless, such flows are of interest since there are many instances in which rotors are surrounded by fluid annuli. Fritz (1970) used an extension of a lubrication theory in which he included fluid inertia and fluid frictional effects for several types of flow in the annulus, including Taylor vortex flow and fully turbulent flow. Though some of his arguments are heuristic, the results are included here because of their practical value. The rotordynamic forces which he obtains are

$$
\begin{align*}
& F_{n}^{*}=\frac{\pi \rho R^{3} L}{\delta}\left[\epsilon\left(\frac{\Omega}{2}-\omega\right)^{2}-\frac{d^{2} \epsilon}{d t^{2}}-\Omega f \frac{d \epsilon}{d t}\right]  \tag{Mce1}\\
& F_{t}^{*}=\frac{\pi \rho R^{3} L}{\delta}\left[\Omega \epsilon f\left(\frac{\Omega}{2}-\omega\right)+(\Omega-2 \omega) \frac{d \epsilon}{d t}\right] \tag{Mce2}
\end{align*}
$$

where $f$ is a fluid friction term that varies according to the type of flow in the annulus. For laminar flow, $f=12 \nu / \delta^{2} \Omega$ and the first term in the square bracket of $F_{t}^{*}$ and the last term in $F_{n}^{*}$ are identical to the forces for a noncavitating long bearing as given in equation (Mcd4). But, Fritz also constructs forms for $f$ for Taylor vortex flow and for turbulent flow. For example, for turbulent flow

$$
\begin{equation*}
f=1.14 f_{T} R / \delta \tag{Mce3}
\end{equation*}
$$

where $f_{T}$ is a friction factor that correlates with the Reynolds number, $R e_{\Omega}$.
The other terms in equations (Mce1) and (Mce2) that do not involve $f$ are caused by the fluid inertia and are governed by the added mass, $M^{*}=\pi \rho R^{3} L / \delta$, which Fritz confirms by experimental measurements. Note that equations (Mce1) and (Mce2) imply rotordynamic coefficients as follows

$$
\begin{align*}
& M=R / \delta \quad ; \quad c=R / \delta \quad ; \quad K=-R / 4 \delta \\
& m=0 \quad ; \quad C=f R / \delta \quad ; \quad k=f R / 2 \delta \tag{Mce4}
\end{align*}
$$

The author also examined these flows using solutions to the Navier-Stokes equations (Brennen 1976). For annuli in which $\delta$ is not necessarily small compared with $R$, the added mass becomes

$$
\begin{equation*}
M^{*}=\frac{\pi \rho L R^{2}\left(R_{S}^{2}+R^{2}\right)}{\left(R_{S}^{2}-R^{2}\right)} \tag{Mce5}
\end{equation*}
$$

where $R_{S}$ is the radius of the rigid stator.

