Perfect Gas

Many gases, especially monatomic gases consisting of single atomic molecules, closely follow the *perfect gas law* at normal temperatures and pressures (though they may deviate significantly in a highly compressed state). The perfect gas law which can be derived from the kinetic theory of gases relates the pressure, density and temperature in a gas by

$$p = \mathcal{R}\rho T \tag{Acd1}$$

where the temperature, T, is the absolute temperature (in degrees Kelvin) and $\mathcal{R}$ is a constant which is proportional to the molecular weight, M, of the gas so that

$$p = \mathcal{R}^* M\rho T \tag{Acd2}$$

where $\mathcal{R}^*$ is known as the universal or ideal gas constant whose value is 8.3144598 in units of $joules/kg\ K^o$ or $m^2/s^2\ K^o$. For air, the most common gas that we shall be dealing with, the effective value of the gas constant is $\mathcal{R} = 280\ m^2/s^2\ K^o$. We also note parenthetically that the kinetic theory of gases leads to a relation between the universal gas constant, Boltzmann's constant, k_B, and Avagadro's constant, N_A, namely $\mathcal{R} = N_A k_B$.

The kinetic theory of gases also leads to the following relations between a increment of temperature and the corresponding increments of internal energy and enthalpy namely

$$de = c_v\, dT \qquad \text{and} \qquad dh = c_p\, dT \tag{Acd3}$$

where c_v and c_p are the specific heats at constant volume and constant pressure respectively with units that are most convenient in fluid mechanics of m^2/s^2. In addition

$$\mathcal{R} = c_p - c_v \tag{Acd4}$$

and denoting the ratio of the specific heats by $\gamma = c_p/c_v$ it follows that

$$c_v = \frac{\mathcal{R}}{(\gamma-1)} \qquad \text{and} \qquad c_p = \frac{\gamma\mathcal{R}}{(\gamma-1)} \tag{Acd5}$$

It is convenient to assume that $\mathcal{R}$, γ, c_v and c_p are all constants for the purposes of our manipulations and calculations though, in fact, they do vary somewhat with thermodynamic conditions.

Since $Tds = dh - dp/\rho$ it follows that in an isentropic process

$$dh = c_p dT = \frac{dp}{\rho} \tag{Acd6}$$

and therefore for a perfect gas for which $\rho = p/\mathcal{R}T$ it follows that

$$\frac{dT}{T} = \frac{\mathcal{R}}{c_p}\frac{dp}{p} = \frac{(\gamma-1)}{\gamma}\frac{dp}{p} \tag{Acd7}$$

Integrating this last equation leads to

$$\ln T = \frac{(\gamma-1)}{\gamma}\ln p + \text{constant} \tag{Acd8}$$

and hence we arrive at the *isentropic relations for a perfect gas*:

$$p \propto T^{\frac{\gamma}{(\gamma-1)}} \quad ; \quad p \propto \rho^{\gamma} \quad ; \quad \rho \propto T^{\frac{1}{(\gamma-1)}} \tag{Acd9}$$