Viscosity

Up to this point in our description of the characteristics of a fluid we have focussed on the properties of a fluid at rest. We now move to discuss the characteristics of a fluid in motion. First we need to describe the property known as viscosity. Though this initial visit to the subject is necessarily somewhat superficial, we will be returning at many points to discuss the complex consequences of this property.
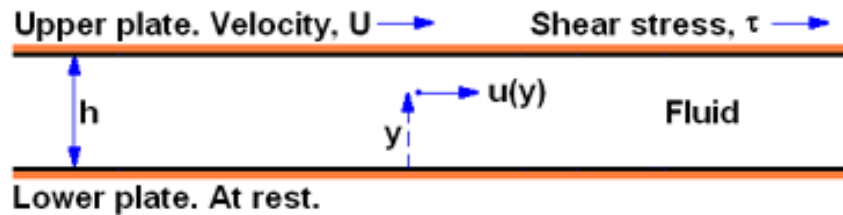

Figure 1: Sketch of Couette flow.

For present purposes it is sufficient to consider the simple flow depicted in Figure 1, namely the flow generated between two infinite, parallel plates, one of which is held fixed (the lower one in the figure) and the other (the upper one) is moving at a velocity, U, in its own plane. The gap between the plates, width h, is filled with a viscous fluid. This is called a Couette Flow or a simple shear flow.

One of the fluid properties that we will discuss in more detail later is the **no-slip condition**. This states that liquid in immediate contact with a solid surface must move with that surface. Physically this comes about because of the intermolecular forces that the solid molecules impose on the liquid molecules adjacent to the surface, essentially locking them to the solid surface. As discussed elsewhere, there are circumstances when this condition no longer holds but it does hold in many flows of practical interest and we will confine the present discussion to such a flow. It follows that the liquid in immediate contact with the lower wall has zero velocity while the fluid in contact with the upper plate is moving with a velocity, U. The question then arises as to the velocity of the fluid at intermediate positions within the gap.

To answer this question rigorously we must consider the forces acting on and within this flow. In order to produce the motion U of the upper plate a force must be applied to it in the direction of motion, parallel with the plate. By symmetry the force that has to be applied to a unit area of the plate is the same everywhere and consequently we define the force applied per unit area as the "shear stress", τ, required to maintain a steady motion and flow. This shear stress is then directly applied by the plate to the fluid in contact with it and that layer of fluid applies the same shear stress to the next layer beneath it and so on all the way across the gap. Thus, at any location within the fluid in the gap the shear stress, τ, is being applied from one layer of fluid to the layer beneath. Therefore, in this particular flow, the shear stress, τ, in the fluid is uniform everywhere.

The effect of a shear stress, τ, at a point within the fluid is to force one layer of fluid to slide over the layer beneath. More specifically, if we denote the distance across the gap by the coordinate, y (see Figure 1), and the fluid velocity parallel to the plates at that location by $u(y)$ then the shear stress, τ, induces a certain velocity gradient, du/dy, in the fluid at that point. The viscosity, μ, is the property of the fluid that is the factor of proportionality between the shear stress, τ, and the velocity gradient:

$$\tau = \mu \frac{du}{dy} \tag{Abd1}$$

Assuming that μ is the same across the gap we can the integrate this equation and apply the boundary conditions (the known velocities, $u = 0$ at $y = 0$ and $u = U$ at $y = h$) to obtain

$$\tau = \frac{\mu U}{h} \quad \text{and} \quad u = U\frac{y}{h} \tag{Abd2}$$

Thus the velocity varies linearly across the gap and the shear stress τ required to maintain an upper plate velocity of U for a gap of h is given by $\mu U/h$. We note that the word "shear rate" is used to refer to a velocity gradient like U/h or du/dy.
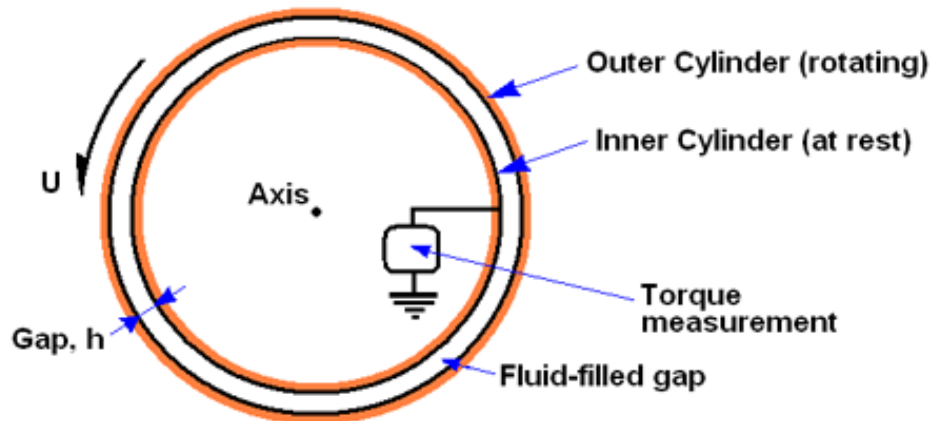

Figure 2: Sketch of a Couette viscometer.

In practice the Couette flow configuration described above is approached in a device known as a Couette viscometer which consists of two cylinders with a fluid-filled gap of width h between them as sketched in Figure 6. The outer cylinder is rotated while the inner is held fixed by a mechanism that also measures the torque from which the shear stress is calculated. Provided the gap is small compared with the radius of the cylinders and provided the axial length is sufficient, the flow will be close to a simple shear flow. Experiments can then be run at different speeds and the measured shear stress plotted against the shear rate, U/h, as illustrated in Figure 3. Clearly the slope of the lines in such a graph is the dynamic viscosity, μ. If all the data lie on a straight line like the blue curve, the viscosity is independent of the shear rate and the fluid is said to be "Newtonian". Many of the fluids one deals with in practice, such as water or air, are Newtonian and this is fortunate for this significantly simplifies the analysis of their flows. However there are other fluids which are non-Newtonian and for which the shear stress is not linearly proportional to the shear rate. Some, such as blood, are "shear-thinning" in that, as the shear rate is increased, the shear stress is less than would be expected if the behavior were linear. In the case of blood, this has a physiological advantage since under physical stress the power required by the heart to pump more blood is less than expected. Conversely. other fluids exhibit a "shear-thickening" behavior in which the increase in the shear stress with increasing shear rate exceeds a linear expectation.

The sketch in Figure 3 assumes that the temperature and pressure of the fluid remain constant and that the results do not change with time. Taking the last issue first, there are some liquids whose viscosity at some specific shear rate will increase with time as the experiment continues. Such fluids are termed "rheopectic". With other liquids, termed "thixotropic" the opposite happens and the viscosity decreases. Fortunately neither of these effects occur with the simple fluids like water and air. But the viscosities of air and water do depend quite strongly on temperature and these variations are illustrated by the data in Figure 4. Note that gases differ from liquids in that the viscosity of a gas increases with temperature
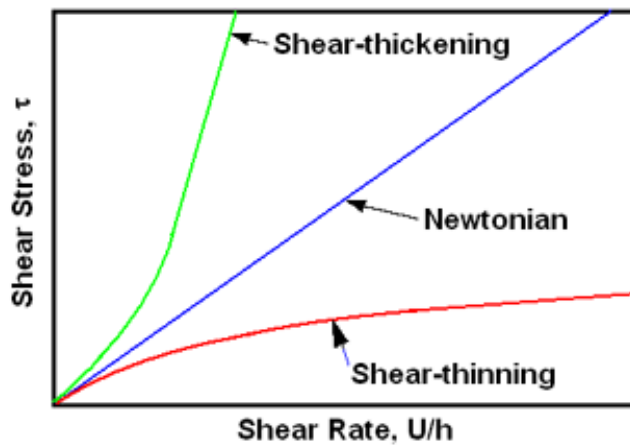
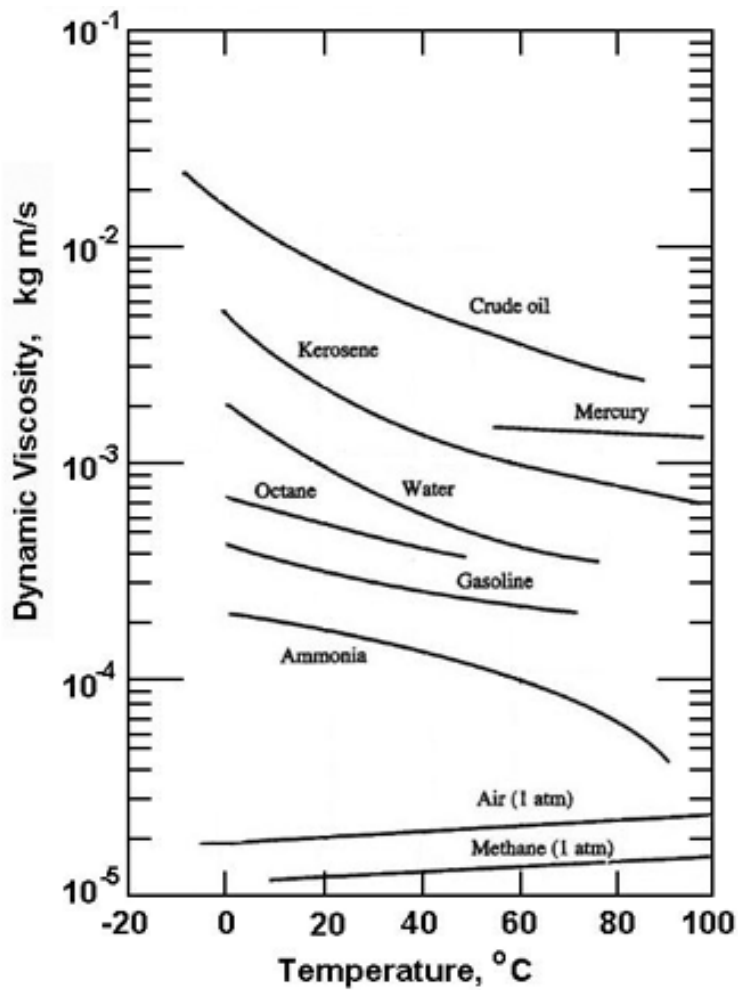Figure 3: Typical results from a Couette viscometer.

Figure 4: The dynamic viscosity of various fluids as a function of temperature.

whereas that of a liquid decreases with temperature. One important everyday consequence is that the oil in your automobile looses its viscosity as the engine overheats.

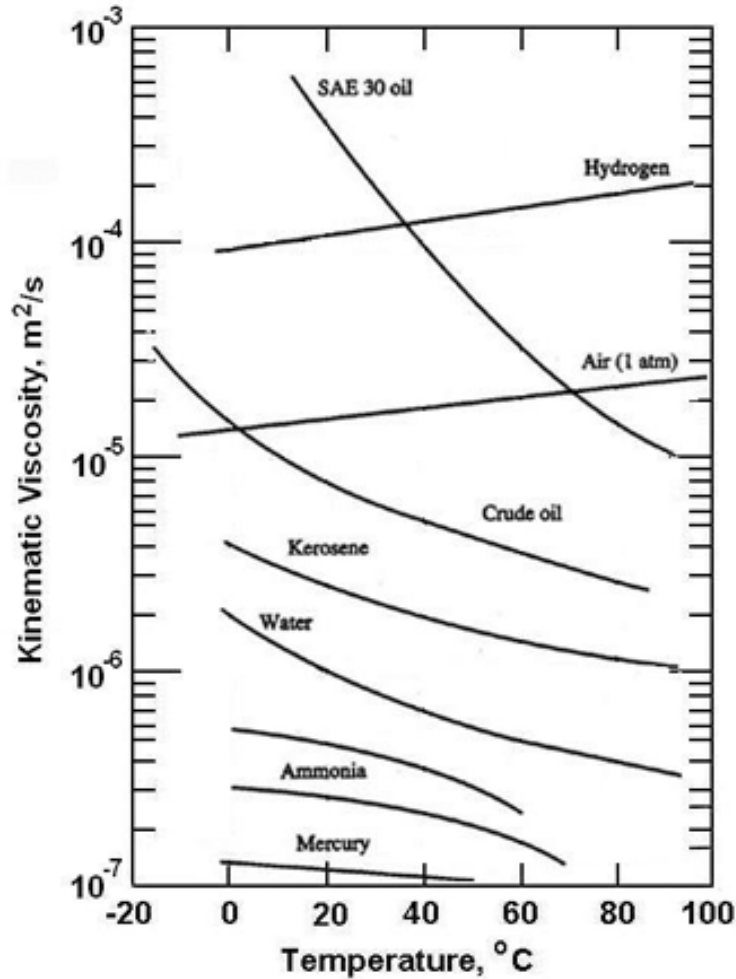In future analyses of some phenomena we will find that features of the flow are dependent on the ratio of

Figure 5: The kinematic viscosity of various fluids as a function of temperature.

the dynamic viscosity, μ, to the density, ρ. This property is called the *kinematic viscosity* and is denoted by $\nu = \mu/\rho$. It has units of m^2/s which are the units of any diffusivity. We shall see that ν is, in fact, the diffusivity that governs the diffusion of vorticity in a laminar fluid flow. The values of ν that correspond to the values of μ shown in Figure 4 are presented in Figure 5. Note how the relative magnitudes of the liquids and gases differ in the two Figures. Whereas the dynamic viscosities of gases are generally much less than those of liquids, the reverse is true of the kinematic viscosities.

Finally, a brief note regarding units. The dynamic viscosity is often quoted in units of *poise* or *centipoise* (*cp* for short); by definition 1 $kg/m\ s$ = 1000 *cp*. Note from Figure 4 that many liquids have dynamic viscosities ranging around $0.1 - 10\ cp$. Moreover the kinematic viscosity is often quoted in units of *stokes* or *centistokes* (*cs* for short) where 1 m^2/s = $10^6\ cs$. Note from Figure 5 that many liquids have kinematic viscosities ranging around $0.1 - 10\ cs$.

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It is appropriate to include a few additional comments on the dependence of the viscosity on the temperature and on the magnitude of the applied stress or strain. As an example we choose to comment on the effective viscosity of the rock in the earth's mantle over very long times in response to buoyancy

forces induced by temperature differences. At higher temperatures (particularly temperatures, T, in a solid or liquid close to the melting temperature, T_M) will depend on the mobility of the atoms or molecules.

In earlier theoretical studies, Gordon (1965, 1967) concluded that the rock in the mantle flowed like a Newtonian liquid because the flow process was one of diffusion creep (Herring-Nabarro creep), in which the stress is indeed simply proportional to the strain rate. This diffusion creep results from the random motions of the molecules and hence the factor of proportionality, or viscosity, is only a function of temperature and not of strain rate. However, Weertman (1970) showed that, at higher temperatures (particularly temperatures, T, close to the melting temperature, T_M) and at high strain rates, the molecular mobility would no longer be isotropic and would depend on the mobility of the atoms or molecules (Figure 6). Under these conditions, the creep would no longer be diffusion creep but would transition to the non-isotropic processes of dislocation creep and dislocation glide. In a related paper this author showed that
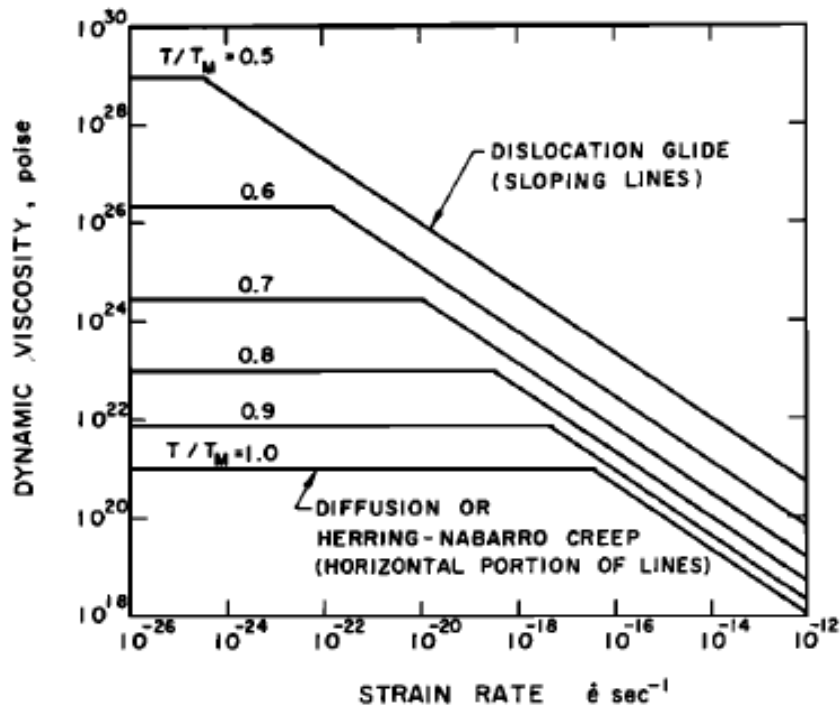

Figure 6: Variation of the mantle viscosity with strain rate, $\dot{e}$, and actual temperature, T, to melting temperature ratio T_M, according to Weertman's (1970) theory.

this non-Newtonian behaviour is consistent with the observations of isostatic recovery of the earth's mantle following the melting of the ice-caps (Brennen 1974).